Algorithmic Approaches to Description Logic (DL) Reasoning
What’s going on?

We want to reason about our models in DL

Hence reasoning algorithms

The algorithms should satisfy three properties:

- Soundness
- Completeness
- Termination
Soundness:
- Every provable statement is true (given true premises)

Completeness:
- Every true statement is provable (from true premises)

Termination:
- Finishes in a finite amount of time
- This can be tricky, algorithms use a variety of different strategies to ensure this property
What's the goal?

As shown previously in the paper, many reasoning tasks boil down to checking knowledge base (KB) satisfiability.

This is what we want our algorithms to do.
Model-theoretic:

- Try to construct a model of a KB
- Success = KB is satisfiable
- Failure = KB is not satisfiable

Proof-theoretic:

- Apply deduction rules to derive further logical statements
- no contradiction = KB is satisfiable
- contradiction = KB is not satisfiable
Algorithm 1 - Tableau

Model-theoretic

Start with a set containing all domain individuals

“Repair” this set until all axioms are fulfilled

Success = KB satisfiable

Will go through an example of this later
Algorithm 2 - Tree-Automata

Model-theoretic

Many DLs satisfy a tree model property

Tree model property + tree-automata = can decide KB satisfiability
Algorithm 3 - Consequence Based Reasoning

Proof-theoretic

Use deduction rules to try to derive new statements

Deduction rules look like this: \[
\frac{\alpha_1 \ldots \alpha_n}{\alpha}
\]

The alphas are statements

If everything above the line is true, then the statement below the line is also true
Algorithm 4 - Resolution

Proof-theoretic

Same principle as consequence based reasoning

Uses one special rule, called the resolution rule

Looks like this:

$$\text{Res} \quad \frac{A_1 \lor \ldots \lor A_i \lor \ldots A_n \quad B_1 \lor \ldots \lor B_j \lor \ldots B_m}{A_1 \lor \ldots \lor A_{i-1} \lor A_{i+1} \lor \ldots A_n \lor B_1 \lor \ldots \lor B_{j-1} \lor B_{j+1} \lor \ldots B_m}$$

Where $A_i$ and $B_j$ are complements of each other