Modeling with Description Logic

- Discusses the added value brought about by “certain DL modeling features”
- Syntactic sugar
  - “Features expressible with stuff you already have”
- Provide insight about model-theoretic consequences that arise from using or not using certain constructs
- … basically a bunch of recipes for basic logical constructs
A lot can be done in ALC

- ALC - Attributive Concept Language with Complements
  - “The prototypical DL”
- Features of ALC
  - Atomic concepts $A, B$
  - Not $\neg C$
  - And $C \cap D$
  - Or $C \cup D$
  - Exists $\exists r. C$
  - For all $\forall r. C$
Concept Disjointness

- “Two concepts C and D are disjoint with respect to an interpretation I, if their extensions do not overlap”
  - Basically means: They have nothing in common
- Formal definitions
  - $C^I \cap D^I = \emptyset$
  - $C \cap D \subseteq \bot$
  - $C \subseteq \neg D$
- Use case → Guarantee that some individual is not an instance of a concept
Domain and Range of Roles

- Given a role, we want statements about the source and target for the respective relation.
  - Domain
    - Role, \( r \) has domain \( C \) in an interpretation \( I \), if any source individual of the relation associated with \( r \), is an instance of \( C \).
    - Definition: \( \exists r. \top \subseteq C \rightarrow \exists \text{authorOf}. \top \subseteq \text{Person} \)
  - Range
    - No intuitive explanation.
    - Definition: \( \top \subseteq \forall r. D \rightarrow \top \subseteq \forall \text{authorOf}. \text{Publication} \)
The Empty Role and Inverses

- The empty role
  - SROIQ has universal and empty concept definitions (\(\top\) and \(\bot\)), but only universal role, \(u\)
  - Empty role missing!
  - New definition: \(\top \subseteq \forall _\text{emptyRole}. \bot\)

- Inverses
  - Inverses allow for traversing roles in reverse direction
    - Can describe individuals with “incoming” roles, as well as “outgoing”
  - Use case → Symmetricity
    - \(r^- \sqsubseteq r \rightarrow \text{marriedWith}^- \sqsubseteq \text{marriedWith}\)
Model Manipulation Part I - Filtration

- “Given a set $C$ of concepts, and an interpretation $I$, we can obtain the filtration of $I$ with respect to $C$, by creating an equivalence relation $\sim$ and letting $\delta \sim \delta'$ if they coincide in terms of concept memberships”

- Basically a super complicated way of saying “grouping by concept”
Model Manipulation Part I - Filtration
Up to Infinity: Cardinality Constraints

- “Create statements about the number of individuals related to a certain individual via a role”
- Should be known from UML and DB-modeling
- 1 to 1, 1 to many, many to many-relationships on roles
- “Value” can also be arbitrary or exact
- Ex: Polygamist $\subseteq \geq 2.\text{Married.}\top$
- Functional roles
  - Roles with at most 1 individual in the target end
  - i.e. hasFather
Model Manipulation Part II: Unraveling

- Unfold a model such that all the parts of the model not containing named individuals are tree-like.
Example

doubleQuaver  ♩

mbt

mbt

crotchet  ♩

 Silent

doubleQuaver  ♩

Silent

crotchet  ♩

Silent

Silent

Silent

Silent

Silent
Far far away: Transitivity

- Examples: ancestorOf, superiorOf, partOf, greaterThan
- Can’t precisely talk about the transitive closure of a given role
Model Manipulation Part III: Disjoint Union

\[ \mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}) \text{ and } \mathcal{J} = (\Delta^{\mathcal{J}}, \cdot^{\mathcal{J}}) \]

\[ \Delta^{\mathcal{I}+\mathcal{J}} = \Delta^{\mathcal{I}} \cup \Delta^{\mathcal{J}} , \quad a^{\mathcal{I}+\mathcal{J}} = a^{\mathcal{I}} , \quad A^{\mathcal{I}+\mathcal{J}} = A^{\mathcal{I}} \cup A^{\mathcal{J}} \]

\[ r^{\mathcal{I}+\mathcal{J}} = r^{\mathcal{I}} \cup r^{\mathcal{J}} \]
Example
Know your Bounds: Nominal Concept and Universal Role

- The modeling power brought about by nominal concepts and universal roles is quite similar
- Capability to bound or fix the number of individuals in the extension of a class or even in the whole domain.
Selfishness

- The self concept enables to speak about “role loops”
- Allows to define concepts based on such situations
Closed/Open World Assumption

- In the *Closed World Assumption* everything in the knowledge base is *true*, everything else is *false*”
  - The knowledge base may be incomplete. The truth of non-derivable axioms is simply unknown.
- DL does *not* make the *Closed World Assumption*

Example of how it works in DL

- f1: All *Ducks* have hats
- f2: Bob is a *Duck*
- KB → Bob wears a hat
- But, can Bob fly?
  - We simple do not know!