Distributed Detection and Localization
A Statistical Signal Processing Approach

Pierluigi Salvo Rossi
Advanced Analytics & Machine Learning Team
Kongsberg Digital AS, Norway
Content

- Introduction

- Sensor Modeling
  - Part I – Detection
  - Part II – Detection and Localization

- (Wireless) Sensor Networks
  - Part I – Distributed Detection
  - Part II – Distributed Detection and Localization
Introduction
Binary Event
Binary Event
Binary Event
Detection
Single Sensor
Distributed Detection
Sensor Network
Distributed Detection
Sensor Network
Distributed Detection
Sensor Network with Fusion Center
Distributed Detection
Sensor Network with Fusion Center
Distributed Detection and Localization Sensor Network with Fusion Center
Distributed Detection and Localization
Sensor Network with Fusion Center
Distributed Detection and Localization Sensor Network with Fusion Center
Distributed Detection and Localization
Sensor Network with Fusion Center
Distributed Detection and Localization
Sensor Network with Fusion Center
Distributed Detection and Localization Sensor Network with Fusion Center
Distributed Detection and Localization
Sensor Network with Fusion Center
Applications

• Collaborative Spectrum Sensing in Cognitive Radio

• Event Detection in Underwater Acoustic Sensor Networks

• Leak Detection and Localization in Oil&Gas production/distribution systems
Sensor Modeling
Part I - Detection
Detection Theory – Decision Theory – Hypothesis Testing

\[ p(y|\mathcal{H}_1) \]

\[ p(y|\mathcal{H}_0) \]

\[ \mathcal{H}_0 \rightarrow y \rightarrow \text{Detector} \rightarrow d \rightarrow \mathcal{H}_1 \]

<table>
<thead>
<tr>
<th>True Hp \ Estimated Hp</th>
<th>( \mathcal{H}_0 )</th>
<th>( \mathcal{H}_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{H}_0 )</td>
<td>Correct Decision</td>
<td>Type I Error (False Alarm)</td>
</tr>
<tr>
<td>( \mathcal{H}_1 )</td>
<td>Type II Error (Missed Detection)</td>
<td>Correct Decision (Detection)</td>
</tr>
</tbody>
</table>

\[ P_D = p(d = \mathcal{H}_1|\mathcal{H}_1) \]

\[ P_F = p(d = \mathcal{H}_1|\mathcal{H}_0) \]

\[ P_M = p(d = \mathcal{H}_0|\mathcal{H}_1) = 1 - P_D \]
Receiver Operating Characteristic (ROC)

\[ p(y|\mathcal{H}_0) \quad p(y|\mathcal{H}_1) \]

Detection Probability

False Alarm Probability

threshold
ROC – Increasing the Signal Power

\[ p(y|\mathcal{H}_0) \quad \text{and} \quad p(y|\mathcal{H}_1) \]

Detection Probability

False Alarm Probability

\[ p(y|\mathcal{H}_0) \quad \text{and} \quad p(y|\mathcal{H}_1) \]

threshold
ROC – Reducing the Noise Power

![Diagram showing ROC curves]

Detection Probability

False Alarm Probability

\[ p(y|\mathcal{H}_0) \]

\[ p(y|\mathcal{H}_1) \]
Sensor Model

True Hp \ Estimated Hp

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{H}_0$</th>
<th>$\mathcal{H}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{H}_0$</td>
<td>Correct Decision</td>
<td>Type I Error (False Alarm)</td>
</tr>
<tr>
<td>$\mathcal{H}_1$</td>
<td>Type II Error (Missed Detection)</td>
<td>Correct Decision (Detection)</td>
</tr>
</tbody>
</table>

$P_D = p(d = \mathcal{H}_1|\mathcal{H}_1)$

$P_F = p(d = \mathcal{H}_1|\mathcal{H}_0)$
Local Test – Optimum Test – Likelihood Ratio Test (LRT)

• LRT is the optimum test in the Neyman-Pearson framework and in the Bayesian framework
  - $y|\mathcal{H}_0 \sim p(y|\mathcal{H}_0)$
  - $y|\mathcal{H}_1 \sim p(y|\mathcal{H}_1)$

• Compute the likelihood ratio or equivalently the log-likelihood ratio (LLR)
  - $\lambda(y) = \ln \left( \frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)} \right)$

• Compare the LLR with a threshold
  - $\lambda(y) \geq \gamma$

• Requires complete knowledge of the conditional probabilities $p(y|\mathcal{H}_0)$ and $p(y|\mathcal{H}_1)$
LRT – Example 1 (Shift in Mean)

• Statistical Signal Model
  \[ y|\mathcal{H}_0 \sim \mathcal{N}(0; \sigma^2) \]
  \[ y|\mathcal{H}_1 \sim \mathcal{N}(\mu; \sigma^2) \]

• Compute the LLR
  \[ \lambda(y) = \ln \left( \frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)} \right) = \frac{\mu}{\sigma^2} y - \frac{\mu^2}{2\sigma^2} \]

• LRT is equivalent to Level Test
  \[ y \gtrless \gamma \]
LRT – Example 2 (Shift in Variance)

- **Statistical Signal Model**
  - \( y|\mathcal{H}_0 \sim \mathcal{N}(0; \sigma_0^2) \)
  - \( y|\mathcal{H}_1 \sim \mathcal{N}(0; \sigma_1^2) \)

\[
p(y|\mathcal{H}_0) = \frac{1}{\sqrt{2\pi \sigma_0^2}} e^{-\frac{y^2}{2\sigma_0^2}}
\]

\[
p(y|\mathcal{H}_1) = \frac{1}{\sqrt{2\pi \sigma_1^2}} e^{-\frac{y^2}{2\sigma_1^2}}
\]

- **Compute the LLR**
  - \( \lambda(y) = \ln \left( \frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)} \right) = \frac{1}{2} \frac{\sigma_1^2 - \sigma_0^2}{\sigma_0^2 \sigma_1^2} y^2 + \frac{1}{2} \ln \left( \frac{\sigma_0^2}{\sigma_1^2} \right) \)

- **LRT is equivalent to Energy Test**
  - \( y^2 \approx \gamma \)
Practical Tests

• (Optimum) LRT

\[ \ln \left( \frac{p(y|H_1)}{p(y|H_0)} \right) \geq \gamma \]

• Test commonly employed in absence of other relevant information
  
  – Level Test

\[ y \geq \gamma \]

  – Energy Test

\[ y^2 \geq \gamma \]
(Wireless) Sensor Networks

Part I – Distributed Detection
Sensor-Network Architecture

Global Test and Global Performance

\[ \lambda \geq \gamma \]

\[ Q_D = p(d = \mathcal{H}_1 | \mathcal{H}_1) \]

\[ Q_F = p(d = \mathcal{H}_1 | \mathcal{H}_0) \]

Local Test and Local Performance

\[ \lambda_k \geq \gamma_k \]

\[ P_{D,k} = p(d_k = \mathcal{H}_1 | \mathcal{H}_1) \]

\[ P_{F,k} = p(d_k = \mathcal{H}_1 | \mathcal{H}_0) \]
Sensor-Network Architecture

• Possible assumptions on the information processing at sensor location
  – **Hard decisions**, local binary decision \( d_k \in \{0,1\} \)
  – **Soft decisions**, level of confidence, multibit quantization of the LRT \( d_k \in \{0,1, \ldots, 2^n - 1\} \)
  – **Analog information**, in the ideal case of infinite precision, LLR information is sent (e.g. \( d_k = \lambda_k(y_k) \))

• Possible assumptions on the reporting channel
  – Perfect channel: \( r_k = d_k \)
  – Parallel Access Channel (**no interference**): \( r_k = f_k(d_k) \)
  – Multiple Access Chanel (**interference**): \( r = f(d_1, d_2, \ldots, d_K) \)
  – MIMO Chanel (**interference and multiple antennas**): \( r_n = f_n(d_1, d_2, \ldots, d_K) \)

  – Common channel models:
    • Binary Symmetric Channel
    • Additive White Gaussian Noise Channel
    • Rayleigh-Fading Channel

• The fusion center takes a global decision depending on a specific **fusion rule**: \( \lambda(r_1, r_2, \ldots, r_N) \geq \gamma \)
MIMO Decision Fusion in WSNs


Why MIMO in WSNs?

• Introduces **spatial diversity**
  – Fading mitigation

• Is **spectrally efficient**
  – Resource saving

• Comes (almost) for **free**
  – Exploiting interference
  – No additional cost except for appropriate processing
System Model

\[ y = Hx + w \]

- \( y = (y_1, \ldots, y_N)^T \) is the **received-signal vector**
- \( y_n \in \mathbb{C} \) is the complex-valued signal received at the \( n \)th RX antenna

- \( H = \begin{pmatrix} (H_{1,1}, \ldots, H_{1,K})^T, \ldots, (H_{N,1}, \ldots, H_{N,K})^T \end{pmatrix}^T \) is the **channel matrix**
- \( H_{n,k} \sim \mathcal{N}(0; 1) \) is the channel coefficient between the \( k \)th sensor and the \( n \)th RX antenna

- \( x = (x_1, \ldots, x_K)^T \) is the **transmitted vector**
- \( x_k \in \{-1, +1\} \) is the BPSK symbol transmitted by the \( k \)th sensor

- \( w = (w_1, \ldots, w_N)^T \) is the **noise vector**
- \( w_n \sim \mathcal{N}(0; \sigma_w^2) \) is the AWGN at the \( n \)th RX antenna

**SNR**

- \( \text{SNR}_{tx} = \frac{K}{\sigma_w^2} \)
- \( \text{SNR}_{rx} = \frac{KN}{\sigma_w^2} \)

**SNR**

- \( \text{SNR}^*_y \geq y \)

- \( Q_D = p(\lambda > y | \mathcal{H}_1) \)

- \( Q_F = p(\lambda > y | \mathcal{H}_0) \)
Performance Benchmarks

• **Observation Bound**: noisy sensing with perfect reporting

\[
Q_F = \sum_{k=g}^{K} (\binom{K}{k}) P_F^k (1 - P_F)^{K-k} \\
Q_D = \sum_{k=g}^{K} (\binom{K}{k}) P_D^k (1 - P_D)^{K-k}
\]

• **Communication Bound**: perfect sensing with noisy reporting

• **Optimal Fusion Rule**: LRT

\[
\lambda(y) = \ln \left( \frac{\sum_{x \in \{\pm 1\}^K} e^{-\frac{||y-Hx||^2}{2\sigma_w^2}} \prod_{k=1}^{K} p(x_k | \mathcal{H}_1)}{\sum_{x \in \{\pm 1\}^K} e^{-\frac{||y-Hx||^2}{2\sigma_w^2}} \prod_{k=1}^{K} p(x_k | \mathcal{H}_0)} \right)
\]

- High computational complexity: exponential with the number of sensors \( \sigma(2^KN) \)
- Numerical instability: large dynamic range is problematic with fixed-point implementations
- Excessive knowledge requirements: local performance, channel matrix, noise variance
Alternative Fusion Rules

- **Decode-and-Fuse approach**
  - **Maximum Ratio Combining**
    (optimum at low SNR, linear complexity $\sigma(N)$, requires partial channel)
  - **Equal Ratio Combining**
    (no optimality, linear complexity $\sigma(N)$, requires less partial channel)
  - **Max-Log**
    (optimal, reduced exponential complexity, full knowledge)

- **Decode-then-Fuse approach**
  - **Chair-Varshney Rule with Maximum Likelihood Estimation**
    (optimum at high SNR, reduced exponential complexity, requires local performance and channel matrix)
  - **Chair-Varshney Rule with Minimum Mean Square Error Estimation**
    (no optimality, polynomial complexity $\sigma(NK^2 + N^2)$, full knowledge)
Sensor Modeling
Part II – Distributed Detection and Localization
Sensor Model

\[ p(y|\mathcal{H}_1) \]

\[ p(y|\mathcal{H}_0) \]

True Hp \ Estimated Hp

<table>
<thead>
<tr>
<th>True Hp \ Estimated Hp</th>
<th>( \mathcal{H}_0 )</th>
<th>( \mathcal{H}_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{H}_0 )</td>
<td>Correct Decision</td>
<td>Type I Error (False Alarm)</td>
</tr>
<tr>
<td>( \mathcal{H}_1 )</td>
<td>Type II Error (Missed Detection)</td>
<td>Correct Decision (Detection)</td>
</tr>
</tbody>
</table>

\[ P_D = p(d = \mathcal{H}_1|\mathcal{H}_1) \]

\[ P_F = p(d = \mathcal{H}_1|\mathcal{H}_0) \]
Sensing Model

\[ y = \theta \cdot g(x; x_T) + w \]

- \( y \) is the measurement at the sensor
- \( w \sim \mathcal{N}(0; \sigma_w^2) \) is the noise at the sensor
- \( x \) is the location of the sensor

- \( \theta \) is the intensity of the target to be detected
  - Unknown and Deterministic: \( \theta \in \Omega_\theta \)
    - e.g. \( \theta \in [-\theta_0, +\theta_0] \)
  - Unknown and Stochastic: \( \theta \sim p(\theta) \)
    - e.g. \( \theta \sim \mathcal{N}(0; \sigma_\theta^2) \)
- \( g(\cdot;\cdot) \) is the (distance-dependent) amplitude attenuation function (AAF) or spatial signature
- \( x_T \) is the target location
Amplitude Attenuation Function (AAF)

- Comes from domain knowledge

- Represents the physical phenomenon and related propagation

- Common AAF with EM signals:
  - **Exponential AAF**
    \[ g^2(x; x_T) = e^{\frac{\|x-x_T\|^2}{\eta^2}} \]
  - **Power-Law AAF**
    \[ g^2(x; x_T) = \frac{1}{1+\frac{\|x-x_T\|^2}{\eta^2}} \]
Local Test

• Statistical Signal Model

- \( y|\mathcal{H}_0 \sim \mathcal{N}(0; \sigma_w^2) \)

\[
p(y|\mathcal{H}_0) = \frac{1}{\sqrt{2\pi \sigma_w^2}} e^{-\frac{y^2}{2\sigma_w^2}}
\]

- \( y|\mathcal{H}_1 \sim \mathcal{N}(0; \sigma_T^2 g^2(x; x_T) + \sigma_w^2) \)

\[
p(y|\mathcal{H}_1) = \frac{1}{\sqrt{2\pi (\sigma_T^2 g^2(x; x_T) + \sigma_w^2)}} e^{-\frac{y^2}{2(\sigma_T^2 g^2(x; x_T) + \sigma_w^2)}}
\]

• Compute the LLR

\[
\lambda(y) = \ln \left( \frac{p(y|\mathcal{H}_1)}{p(y|\mathcal{H}_0)} \right) = \frac{\Gamma_s}{2} \frac{g^2(x; x_T)}{\sigma_T^2 g^2(x; x_T) + \sigma_w^2} y^2 + \frac{1}{2} \ln \left( \frac{1}{1 + \Gamma_s g^2(x; x_T)} \right)
\]

\[
\Gamma_s \triangleq \frac{\sigma_T^2}{\sigma_w^2} \quad \text{sensing SNR}
\]

• LRT is equivalent to Energy Test

- \( y^2 \gtrsim \gamma \)
Local Performance

- Assume fixed local FA probability

- Assume fixed AAF

- Evaluate local detection probability vs target distance

\[ P_F = 2Q \left( \frac{\gamma}{\sigma_w^2} \right) \]

\[ P_D = 2Q \left( \frac{\gamma}{\sigma_T g^2(x; x_T) + \sigma_w^2} \right) \]
Local Performance

- Assume fixed local FA probability
- Assume fixed AAF
- Evaluate local detection probability vs target distance
- Performance improves with sensing SNR

\[
P_F = 2Q\left(\frac{\gamma}{\sigma_w^2}\right)
\]
\[
P_D = 2Q\left(\frac{\gamma}{\sigma_T^2 g^2(x;x_T)+\sigma_w^2}\right)
\]
Local Performance (ROC)

- Performance worsens with distance
- Performance improves with sensing SNR

\[ P_F = 2Q\left(\frac{Y}{\sigma_w^2}\right) \]
\[ P_D = 2Q\left(\frac{Y}{\sigma_T^2 g^2(x|x_T)+\sigma_w^2}\right) \]
(Wireless) Sensor Networks

Part II – Distributed Detection and Localization
MIMO Decision Fusion in WSNs

Sensor-Network Architecture

Global Test and Global Performance

\[ \lambda \geq \gamma \]

\[ Q_D = p(d = H_1 | H_1) \]

\[ Q_F = p(d = H_1 | H_0) \]

Local Test and Local Performance

\[ \lambda_k \geq \gamma_k \]

\[ P_{D,k} = p(d_k = H_1 | H_1) \]

\[ P_{F,k} = p(d_k = H_1 | H_0) \]
Counting Rule (CR)

• Simple and intuitive strategy is to count the number of reported detections
  – \( \lambda = \sum_{k=1}^{K} d_k \)

• Advantages
  – System knowledge not required (e.g. local performance, sensing SNR, etc.)
  – It is optimal in the case of homogeneous sensor networks
    \( P_{F,k} = P_F \) and \( P_{D,k} = P_D \)

• Disadvantages
  – Poor performance in practical scenarios of interest
  – No localization provided
Ring Scenario

![Diagram of ring scenario](image-url)
Performance of CR in Ring WSNs

• Assume a WSN with K sensors

• All sensors have the same distance from the target

• Performance improves with K

• Unrealistic assumption
  – if present the target is in known position
  – good approximation for large spreading factors
Performance of CR in Ring WSNs

- Assume a WSN with K sensors
- All sensors have the same distance from the target
- Performance improves with K
- Performance improves with sensing SNR

- Unrealistic assumption
  - if present the target is in known position
  - good approximation for large spreading factors
Randomly-Deployed Sensors
Performance of CR in Random WSNs

• Assume a WSN with K sensors

• Sensors are randomly generated in the sensor area

• Target (if present) is randomly generated in the target area

• Performance improves with sensing SNR

• Performance improves with $\eta$
Optimum Rule – (Clairvoyant) LRT

• Compute the LLR
  \[ \lambda = \ln \left( \frac{p(d|H_1)}{p(d|H_0)} \right) = \sum_{k=1}^{K} d_k \ln \left( \frac{P_{D,k}}{P_{F,k}} \right) + (1 - d_k) \ln \left( \frac{1 - P_{D,k}}{1 - P_{F,k}} \right) \]

• Advantages
  – Optimum performance

• Disadvantages
  – Cannot be implemented in practice
    Requires knowledge of both \( P_{F,k} \) and \( P_{D,k} \) which is unrealistic (because depending on \( x_T \) and \( \sigma_T^2 \))
Generalized LRT (GLRT)

• Compute the LGLR using ML estimation

\[
\lambda = \ln \left( \frac{\max_{x_T; \sigma_T^2} p(\mathbf{d}|\mathcal{H}_1; x_T; \sigma_T^2)}{p(\mathbf{d}|\mathcal{H}_0)} \right) = \sum_{k=1}^{K} \left[ d_k \ln \left( \frac{P_{D,k}(\hat{x}_T; \sigma_T^2)}{P_{F,k}} \right) + (1 - d_k) \ln \left( \frac{1-P_{D,k}(\hat{x}_T; \sigma_T^2)}{1-P_{F,k}} \right) \right]
\]

\[
(\hat{x}_T; \hat{\sigma}_T^2) = \arg \max_{x_T; \sigma_T^2} p(\mathbf{d}|\mathcal{H}_1; x_T; \sigma_T^2)
\]

• Advantages
  – System knowledge not required (e.g. local performance, sensing SNR, etc.)
  – Excellent performance for both detection and localization tasks

• Disadvantages
  – Requires optimization procedure for ML estimation (e.g. grid search)
Randomly-Deployed Sensors
Performance of GLRT and CR in Random WSNs

- Sensors are randomly generated in the sensor area
- Target (if present) is randomly generated in the target area
- Performance improves with sensing SNR
- The improvement of GLRT wrt CR is apparent

![Graph showing performance comparison between GLRT and CR](image)

*estimated SNR_s = 10.16 dB*
Alternative Fusion Rules

• Bayesian approach
  – Bayesian LLR

• Locally Optimum Detection (LOD) approach
  – Generalized LOD (GLOD)

• Hybrid approach
  – Bayesian/GLLR
  – Bayesian/LOD
WORLD CLASS
THROUGH PEOPLE, TECHNOLOGY AND DEDICATION