Prognostics and health management of safety-instrumented systems
— Approaches of degradation modeling and decision-making

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Research motivation
Risk?

Risk assessment involves considering four main strategies: Transfer, Avoidance, Acceptance, and Reduction/mitigation. Each strategy is represented on a grid with axes for Likelihood and Consequence.

- **Transfer**: Moving the risk to another party.
- **Avoidance**: Eliminating the risk altogether.
- **Acceptance**: Accepting the risk without taking any action.
- **Reduction/mitigation**: Reducing the likelihood or consequence of the risk.

Examples of significant incidents:

- **Deepwater Horizon oil spill, 2010**
- **Fukushima Daiichi Accident, 2011**

* https://www.google.com
Layer of protection

Example of safety barriers in process industry

High-Integrity Pressure Protection System
Safety-instrumented systems (SISs)

Operational modes and failure modes of SISs

- Dormant State
  - demands
  - Proof test

- Active state

- State = unknown
  - No DU fault
  - revealed

- State = As-good-as-new
  - repair
  - DU fault

- Safe detected
  - fail-to-function
  - $\gamma_e$?

- Safe undetected

- Dangerous detected

- Dangerous undetected

- Automatic self-test

- Proof test

- Dangerous failure
Performance measurement – Binary states

PFD_{avg}: average probability failure on demand in each test interval is used to quantify the reliability of SIS.

\[ PFD_{avg} = \frac{1}{\tau} \int_{0}^{\tau} PFD(t) \, dt \]

Failure probability at time \( t \)

Test interval
Research questions
Testing and maintenance policy

Time-based*
- The predefined proof tests
- Binary system state
- Failure rate-based assessment methods

Performance-based
- Performance-based updating tests
- Time-dependent performance
- Degradation modeling

Gap analysis

Key steps to bridge gap
1. System degradation modeling
2. Decision-making for the upcoming tests with collected information.

IEC61511 Functional safety - Safety instrumented systems for the process industry sector
Prognostics and health management (PHM)

PHM can be used for:
1. evaluating the reliability of systems of their life cycle;
2. determining the possible occurrence of failures and risk reduction;
3. highlighting the residual useful lifetime (RUL) estimation.

Benefits and challenges

Benefits
1. Advance warning of failures and maintain the required function;
2. Avoid unnecessary tests;
3. Optimized maintenance;
4. Logistic support and cost reduction.

Challenges
1. Degradation modeling;
2. Redundancy structure in degradation modeling;
3. Time dependent measurement of SISs;
4. Decision-making within the required SIL.

* Article I:
Influencing factor of SIS

- Working condition
  - Aging
  - Required function

- Random demands
  - Demand rate
  - Damage size

- Structure
  - Redundancy
  - Heterogeneity

- Testing and maintenance
  - Testing strategy
  - Maintenance strategy
  - Imperfectness
Time-dependent Performance

Binary state VS time-dependent performance
Research questions and objectives

Degrading performance
- Continuous aging
- The required performance

Redundancy structure
- Same damage
- Only the activated ones

Evaluation criteria
- Condition-based maintenance
- Economics

- Continuous aging on time-dependent SISs degrading performance;
- Hybrid effects of continuous aging and random demands;
- Assessment method considering the effectiveness of collected information in tests
- Balancing SIS performance and economic targets in decision-making
Contribution
Contribution

— Degrading performance

Objective: Continuous aging on time-dependent SISs degrading performance

Method: Stochastic process

Output:
Article III:

Article IV:
Degrading performance

1. Time dependent state: working, degraded and failed;

<table>
<thead>
<tr>
<th>State</th>
<th>Status</th>
<th>State description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Working</td>
<td>The system is functioning as specified</td>
</tr>
<tr>
<td>1</td>
<td>Degraded</td>
<td>The system has a degraded performance but still functioning</td>
</tr>
<tr>
<td>2</td>
<td>Failed</td>
<td>The system has a failed fault</td>
</tr>
</tbody>
</table>
2. Periodic proof test with interval $\tau$;
3. Different maintenance strategies are taken based on the state of component
   - Failed state: corrective maintenance (AGAN)
   - Degraded state: imperfect preventive maintenance ($\omega_bL$)
   - Working state: no maintenance

Research questions:
- System performance ?
- Conditional PFD($t$)
- $\omega_aL$, $\omega_bL$ ?
Degrading performance

Degradation process: homogenous Gamma degradation process
Maintenance: only at proof test date

- $A(t) = \Pr(X(t)<L)$
- The conditional $A(t)$: $A(t) = \Pr(X(t)<L|X(\tau) = \mu)$

$$PFD_{\text{avg}} = \frac{1}{\tau} \int_{0}^{\tau} [1 - A(t)] dt$$

- Degradation level $X(t)$ accumulates with time;
- $A(t)$ reduces;
- $PFD_{\text{avg}}$ increases with time.
Degrading performance

Conclusion:

- System $PFD_{avg}$ increases with time even working at tests.
- $PFD_{avg}$ is more susceptible to the degree of degradation initiating a PM;
- The theoretical basis for the updating testing interval given SIL.
Contribution

—— Redundant structure

**Objective**: Hybrid effects of continuous aging and random demands

**Method**: Stochastic process + Poisson process

**Output**:

Article II:

Article VI:
Degradation process of single unit:

- Continuous aging;
- Random demands

System reliability of such a 1oo2 by time $t$ is the probability that total degradation of at least one unit is less than the threshold $L$, as,

$$R_S(t) = \Pr(\{Z_1(t) < L_1\} \cup \{Z_2(t) < L_2\})$$
Performance analysis

1. Degradation processes of one unit:
   - Continuous aging process: homogeneous gamma process
   - Random demands: Poisson process with rate $\lambda_{de}$
   - Demand damage: Gamma distribution

2. For 1002, random demands will have same damage effects on two units.
3. Two components are dependent due to the same damage caused by random demands.
Performance analysis

1oo2 System performance: \( R(t) \) and conditional PFDavg

- System is quite reliable at beginning;
- System reliability will be overestimated if only the aging process is considered.
- System conditional PFD\(_{\text{avg}}\) increases with time.
- Considering aging and damage caused by random demands can make the system reliability and PFD\(_{\text{avg}}\) stricter than only aging process.
Performance analysis

- Same working conditions
- Demand only on the ‘effective activated one’.

Binomial distribution of getting $k$ demands in total $n$ demands on unit 1

$$f(k, n, p) = Pr(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$
Performance analysis

1. \( R(t) \) is quite high at beginning;
2. System \( R(t) \) and MTTF reach a minimum value when \( p=0.5 \), a maximum value with \( p=0 \).
3. The optimal strategy: \textbf{one unit as standby until the primary one failed}. 

\( R(t) \) and MTTF of 1oo2 system with activation probability \( p \) of unit 1
2oo3 configuration performance

$$p_1 = \text{Pr}(\text{Activating unit 1})$$

$$p_2 = \text{Pr}(\text{Activating unit 2}|p_1)$$

1. System MTTF reaches the minimum when $p_1 = 0$, also when $(p_1, p_2) = (1,0)$ and $(p_1, p_2) = (1, 1)$;
2. System performance reaches the worst state while keeping the fixed combinations for all demands;
3. For 2oo3 configuration, demands should be arranged equally to each unit.
Contribution
— Decision-making approach

**Objective**: Assessment method considering the effectiveness of collected information in tests
Balancing SIS performance and economic targets in decision-making

**Method**: Markov process

**Output**:

Article V:
Discrete degradation — 1001 configuration

<table>
<thead>
<tr>
<th>Notation</th>
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</tr>
</thead>
<tbody>
<tr>
<td>W</td>
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</tr>
<tr>
<td>D</td>
<td>Degraded</td>
</tr>
<tr>
<td>F</td>
<td>Failed</td>
</tr>
</tbody>
</table>

Imperfect degraded state revealing

- Degradation is not observed directly
- Inaccurate threshold setting for the state
- Subjective errors
Discrete degradation — 1001 configuration

$\alpha = \text{Pr(Degradation is detected in a proof test} | \text{Degradation has occurred})$

Testing and maintenance matrix:
- PM for the degraded state
- CM for the failed state

\[
A = \begin{pmatrix}
1 & 0 & 0 \\
\alpha & 1 - \alpha & 0 \\
1 & 0 & 0
\end{pmatrix}
\]

1. When $\alpha=1$, the degraded state will be repair, $\text{PFD}(t)$ keeps the same in each test interval.
2. When $\alpha=0$, no degraded state will be repair, $\text{PFD}(t)$ increases each test interval.
Discrete degradation — 1oo2 configuration

Testing and maintenance strategy for 1oo2 configuration

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Simultaneous testing</th>
<th>Strategy details</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>PM and CM for the degraded and failed state, respectively.</td>
<td></td>
</tr>
</tbody>
</table>
| II       | Staggered testing    | • PM and CM for the tested unit  
|          |                      | • No action on the other |
| III      | Staggered testing    | • PM and CM for the tested unit  
|          |                      | • When CM, perform a replacement on the other |
Discrete degradation — 1oo2 configuration

1. System PFDavg independents with \((\alpha_1, \alpha_2)\) in first test interval \((0, \tau)\).
2. System PFDavg keeps a constant value with \(\alpha_1 = \alpha_2 = 1\).
3. System PFDavg increases with time when \(\alpha_1 \neq 1, \alpha_2 \neq 1\).
Discrete degradation — 1oo2 configuration

\[ k_{ji} = \frac{\text{PFDavg with strategy } j}{\text{PFDavg with strategy } i} \]

Strategy III > Strategy II > Strategy I
Discrete degradation — 1oo2 configuration

Expected cost in single test interval

\[ EC_i = EC_{PT} + EC_{PM} + EC_{CM} \]

Linked with revealing coverage \( \alpha \)

One-time installation cost per unit

\[ LCC = C_0 + \sum_{i=1}^{n} EC_i \]

Strategy I  
Strategy II  
Strategy III

Strategy III > Strategy I > Strategy II
Discrete degradation — 1oo2 configuration

Selection procedure for optimal testing and maintenance strategy

- PFDavg
- SIL
- LCC

Strategy III
Strategy II
Strategy I

LCC

PFDavg

Low
Medium
High

Low
Medium
High
Concluding remarks
Conclusions

1. The proposed stochastic process-based degradation model provide an advantage of calculating the conditional system performance based on the collected information in tests;

2. Quantitative degradation models are proposed for single-unit and redundant structure systems, to address several factors, as aging, operational history and configuration;

3. A performance-based maintenance framework is proposed to evolve the maintenance scheme.

4. Algorithms are proposed to coordinate system performance and maintenance cost, which provides the quantitative references in the decision-making step of PHM on SISs.
Acknowledgements

- Supervision team: Yiliu, Anne, Elias, and Tieling (UOW);
- Co-authors in these publications;
- RAMS colleagues and friends;
- Families.
Thank you!
Gamma process

Properties of homogeneous gamma process \( \Gamma(t; \alpha; \beta) \):
An homogeneous gamma process with shape parameter \( \alpha \) and scale parameter \( \beta \), is a stochastic process \( X(t); t > 0, \alpha; \beta > 0 \)

1. \( X(0) = 0 \);
2. \( X(t); t > 0 \) is a stochastic process with independent increments;
3. for \( s < t \), the distribution of the random variable \( X(t) - X(s) \) is the gamma distribution
the increment degradation $X$ for $t - s$, $X(t - s)$ follows a Gamma PDF

$$\Delta X(t - s) \sim \Gamma(\alpha(t - s), \beta) = f_{\alpha(t-s),\beta}(x)$$

$$= \frac{\beta^{\alpha(t-s)}}{\Gamma(\alpha(t-s), 0)} x^{\alpha(t-s)-1} e^{-\beta x}, \alpha, \beta > 0$$

2 total degradation $X(t)$ at time $t$ is less than $x$, $F_X(x, t)$, can be derived as:

$$F_X(x, t)(t) = P\{X(t) < x\} = \int_0^x f_{\alpha t, \beta}(z)dz = \frac{\gamma(\alpha t, x \beta)}{\Gamma(\alpha t)}$$

3 the mean and variance of $X(t)$ are $\frac{\alpha}{\beta} t$ and $\frac{\alpha}{\beta^2} t$, respectively.
System reliability and conditional PFDavg– 1oo2 (Article II)

\[ R_S(t) = \text{Pr}(\{Z_1(t) < L_1\} \cup \{Z_2(t) < L_2\}) \]

In this example, such a 1oo2 SIS needs to meet SIL3. Here, we take different thresholds \( L \) in Fig. 9 as an example. Values of the two variables are at first set as \( \lambda_{du} = 2.5 \times 10^{-5} \), and \( \xi = 4 \) respectively. Similar to Eq. 18, we can connect reliability and average PFD in a test interval

\[ \text{PFD}_{\text{avg}} = 1 - \frac{1}{t-t_0} \int_{t_0}^{t} \frac{R(t)}{R(t_0)} dt \]  \hspace{1cm} (21)

The average value of PFD_1(t) in the first proof test interval \((0, \tau)\) can be obtained then

\[ \text{PFD}_{\text{avg}} = \frac{1}{\tau} \int_0^{\tau} \text{PFD}_1(t) dt = 1 - \frac{1}{\tau} \int_0^{\tau} R(t) dt \]  \hspace{1cm} (13)

Using the survivor function of the system \( R(t) \) in (11), we can get

\[ \text{PFD}_{\text{avg}} = 1 - \frac{1}{\tau} \int_0^{\tau} R(t) dt \]

\[ = 1 - \frac{1}{\tau} \int_0^{\tau} \left( 1 - \frac{2(\nu(t) \cdot \xi)}{(\nu(t)} \right)^2 \cdot e^{-\lambda_{du} t_0} \]

\[ \sum_{k=1}^{\infty} \int_0^{\tau} \left[ 1 - \frac{2(\nu(t) \cdot \xi)}{(\nu(t)} \right]^2 \frac{\nu^{kn} \cdot \xi^{k-1}}{k!} \cdot e^{-\xi t} \cdot \frac{\xi}{k!} dt \]  \hspace{1cm} (14)

A proof-test will be executed at time \( \tau \). If the subsystem is functioning at \( \tau \) with unknown degradation level, \( \text{PFD}_2(t) \) becomes the conditional probability of failure with \( t > \tau \) given functioning by \( \tau \)

\[ \text{PFD}_2(t) = \text{Pr}[T < \tau(t) > \tau, t > \tau] = 1 - \text{Pr}[T > \tau(t) > \tau, t > \tau] \]

\[ = 1 - \frac{\text{Pr}[T > \tau(t)] \cdot \text{Pr}[T > \tau]}{\text{Pr}[T > \tau]} = 1 - \frac{R(t)}{R(\tau)} \]  \hspace{1cm} (15)

The \( \text{PFD}_{\text{avg}} \) in the second test interval \((\tau, 2\tau)\) is then:

\[ \text{PFD}_{\text{avg}} = \frac{1}{\tau} \int_\tau^{2\tau} \text{PFD}_2(t) dt \]

\[ = \frac{1}{\tau} \int_\tau^{2\tau} \left[ 1 - \frac{R(t)}{R(\tau)} \right] dt \]

\[ = 1 - \frac{1}{\tau} \int_\tau^{2\tau} \frac{R(t)}{R(\tau)} dt \]  \hspace{1cm} (16)