Problem 1

a) Two point charges $Q$ have a distance $a$ from each other (see figure below). Use Coulomb’s law to find the force between them.

\[
\begin{array}{c}
\text{Q} \\
\text{a} \\
\text{Q}
\end{array}
\]

b) We add another point charge, $-Q$, so that they form an equilateral triangle with side length $a$ (see figure below). Find the force (magnitude and direction) acting on the charge $-Q$.

\[
\begin{array}{c}
\text{Q} \\
\text{a} \\
\text{a} \\
\text{Q} \\
\text{a} \\
\text{Q}
\end{array}
\]

c) Imagine now that we remove the charge $-Q$ at the top of the triangle and replace it with an infinitely small charge $q$. What is the electric field $E$ perceived by this charge? Why is it important to assume that $q$ is infinitely small when we want to find the field from the two charges $Q$ in task (a)?

Problem 2

We have a charge $Q$ and draw a spherical surface with radius $r$ around it (see figure below).
a) We want to find the \( \mathbf{E} \)-field flux through the surface of the sphere. Show that the flux can be expressed by

\[
\oint \mathbf{E} \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^\pi \frac{Q}{4\pi \varepsilon_0 r^2} r^2 \sin \phi \, d\phi \, d\theta,
\]

and solve this to find the flux.

b) The field from the charge \( Q \) is constant along the surface of the spherical cap, and is always pointing radially outwards (we can see this from the Coulomb force); thus it is parallel with the surface elements \( d\mathbf{S} \). Use this to solve the equation above in a simplified way.

c) Find the flux using the divergence theorem, and by that prove Gauss’ law.

*Tip:* Look at the deduction of Gauss’ law from the notes, compendium or book!

d) Now, let’s assume that the charge \( Q \) is evenly distributed inside the volume of the sphere. Thus we have the charge density

\[
\rho = \frac{Q}{\frac{4}{3} \pi a^3},
\]

inside the sphere, where \( a \) is the radius. Find the spatial electric field \( \mathbf{E} \) inside and outside the sphere.

**Problem 3**

Given two infinitely large planes with the surface charge densities \( \rho_s \) and \( -\rho_s \). Find the spatial electric field.

**Problem 4**

a) Imagine that you have a disc with radius \( a \) that has a constant surface charge density \( \rho_s \) (see the left figure below). Show that the potential \( V \) at a height \( z \) above the center of the disc is given by

\[
V(z) = \frac{1}{4\pi \varepsilon_0} \int_{\text{disk}} \frac{\rho_s dS}{R},
\]

by first finding the potential from a sum of point charges using superposition. Let your point of reference be infinity and assume \( z > 0 \).

b) \( R \) represents the distance between a point on the disc and an observation point on the \( z \)-axis. Find an expression for \( R \) given by \( r \) (the distance between the origin and the point on the disc) and \( z \) (the height of the observation point). Find an expression for \( dS \) given by \( r \) and use these expressions to solve the integral above so that you find:

\[
V(z) = \frac{\rho_s}{2\varepsilon_0} \left( \sqrt{z^2 + a^2} - z \right).
\]

c) Use your results from the previous task to find the electric field \( \mathbf{E} \) for the same point.

*Hint:* \( \mathbf{E} = (0,0,E_z) \) due to symmetry.

d) Find the electric field \( \mathbf{E} \) in the limits \( z \ll a \) and \( z \gg a \). Interpret the results physically.
Problem 5

By using the results from the previous task, find $\mathbf{E}$ at a height $z$ above an infinitely large plane with a hole with a radius $a$ (see the right figure above). The plane has a constant surface charge density $\rho_s$.

*Hint:* Superposition!