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TFE4120 Electromagnetics - Preliminary course

Solution proposal, exercise 5

Problem 1

Ampére's law, $\oint_C \mathbf{H}\cdot d\mathbf{l} = I_{\text{through S}}$, used on a circular loop through the core of the toroid gives

$$H \cdot 2\pi a = NI. \tag{1}$$

We have got $B = \mu H$ which yields

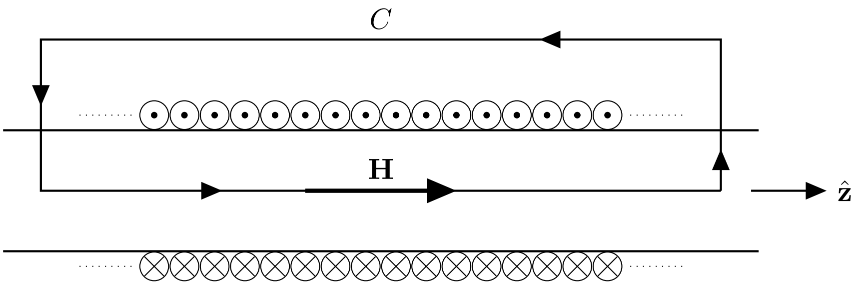
$$\mathbf{B} = \frac{\mu N I}{2\pi a}\hat{\boldsymbol{\varphi}}. \tag{2}$$

For the two instances we find

a) $B \approx 2\mu\text{T}$,

b) $B \approx 10\text{mT}$.

Problem 2



The figure shows a solenoid with N turns distributed on the length l, and the direction of the **H**-field.

a) The **H**-field inside the winding is given by

$$\mathbf{H} = H\hat{\mathbf{z}} = \frac{NI}{l}\hat{\mathbf{z}}. \tag{3}$$

From the correlation $\mathbf{B} = \mu\mathbf{H}$ it can be rewritten into

$$B = \mu\frac{NI}{l}, \tag{4}$$

where $\mathbf{B} = B\hat{\mathbf{z}}$. Let a be the radius of the solenoid.

First, we find the magnetic flux Φ_{cs} through the cross section of the solenoid.

$$\Phi_{\text{cs}} = \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = \int_S B\mathrm{d}S = B \int_S \mathrm{d}S = \mu \frac{NI}{l} \cdot \pi a^2. \tag{5}$$

Since this is the flux through each of the N turns, the total flux Φ_{tot} for the whole solenoid is $\Phi = N\Phi_{\text{cs}}$. Thus, the self inductance is

$$\underline{\underline{L = \frac{\Phi}{I} = \frac{N\Phi_{\text{cs}}}{I} = \frac{\mu \pi a^2 N^2}{l}}}. \tag{6}$$

b) From (6) we see that since the inductance is proportional to N^2, the inductance will quadruplicate when the number of turns is doubled.

c) If the current in the current in the coil is reduced linearly so that $I(0) = I_0$ og $I(\tau) = 0$ we have

$$I(t) = I_0 \left(1 - \frac{t}{\tau}\right), \text{ for } 0 \leq t \leq \tau. \tag{7}$$

Since the induced electromotive force is given by

$$e(t) = -L\frac{\mathrm{d}I(t)}{\mathrm{d}t}, \tag{8}$$

we get

$$\underline{\underline{e(t) = -L\frac{\mathrm{d}}{\mathrm{d}t}I_0\left(1 - \frac{t}{\tau}\right) = \frac{LI_0}{\tau}}}. \tag{9}$$

for $0 \leq t \leq \tau$

Inside the vacuum cleaner there is an electric motor that runs a fan. When we unplug it the current will be reduced during a very short time. From (9) we can see that if the current is reduced rapidly, τ is small, so that the induced voltage is great. Due to the self inductance in the electric motor, there will be large induced voltage. If the voltage difference between the plug and the outlet is great enough, we might have a flashover.

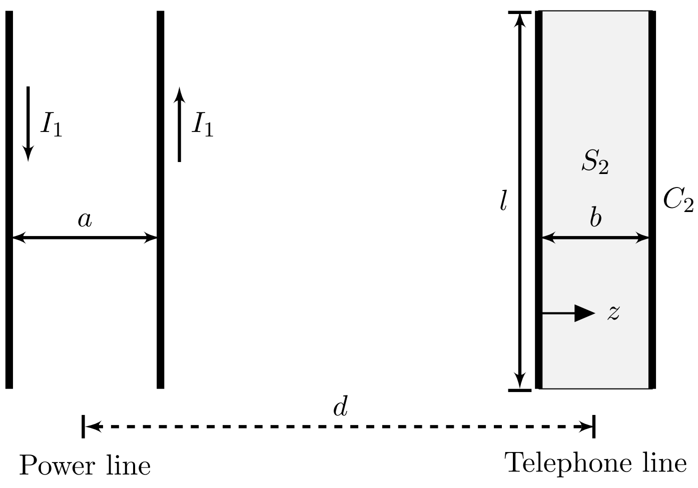
Problem 3

a) The mutual inductance L_{12} is given by

$$\Phi_{12} = L_{12} I_1. \tag{10}$$

where Φ_{12} is the magnetic flux through a loop C_2 due to the current I_1 through the loop C_1. It can be shown that $L_{12} = L_{21}$[1] The loop C_2 and its surface S_2 is defined as in the figure above. We can imagine that the power line and the telephone line creates two closed loops (closed by a load in one end and a generator in the other). By using

[1] This means that it is possible to find $L_{12} = L_{21} = \Phi_{21}/I_2$ by calculating the flux Φ_{21} through the power line when there is a current I_2 in the telephone line. This is more difficult to calculate since we have to use Biot-Savart's law to find the field from the telephone line, and we have to calculate the flux $\Phi_{21} = \int_{S_1} \mathbf{B} \cdot \mathrm{d}\mathbf{S}$ explicitly, since $\mathbf{B}$ no longer is constant.



Ampére's law we find that the magnetic field at a distance r from a long, thin conductor that conducts a current I_1 is given by

$$\oint_{C_1} \mathbf{H} \cdot \mathrm{d}\mathbf{l} = 2\pi r H = I_1, \tag{11}$$

which yields

$$B = \mu_0 H = \frac{\mu_0 I_1}{2\pi r}. \tag{12}$$

The magnetic field inside C_2 caused by the power line consists of a contribution B_1 from the conductor in position $z_1 = -d - \frac{a}{2} + \frac{b}{2}$, and a contribution B_2 from the conductor in position $z_2 = -d + \frac{a}{2} + \frac{b}{2}$ (We have decided to define $z = 0$ at the left conductor of the telephone line). The direction of the magnetic field is orthogonal to the surface encircled by C_2. Thus we can express the magnetic field as

$$\begin{aligned} B(z) &= B_1(z) + B_2(z) \\ &= \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_1}{2\pi r_2} \\ &= \frac{\mu_0 I_1}{2\pi} \left(\frac{1}{z - z_1} - \frac{1}{z - z_2} \right). \end{aligned} \tag{13}$$

Accordingly, the magnetic flux within C_2 is given by

$$\begin{aligned} \Phi_{12} &= \int_{S_2} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = l \int_0^b B(z) \mathrm{d}z \\ &= \frac{\mu_0 I_1 l}{2\pi} \int_0^b \left(\frac{1}{z - z_1} - \frac{1}{z - z_2} \right) \\ &= \frac{\mu_0 I_1 l}{2\pi} \left[\ln\left(\frac{b - z_1}{-z_1} \right) - \ln\left(\frac{b - z_2}{-z_2} \right) \right], \end{aligned} \tag{14}$$

which yields

$$\underline{\underline{L_{12} = \frac{\Phi_{12}}{I_1} = \frac{\mu_0 l}{2\pi} \left[\ln\left(\frac{b - z_1}{-z_1} \right) - \ln\left(\frac{b - z_2}{-z_2} \right) \right]}}. \tag{15}$$

b) From Faraday's law we have

$$e = -\frac{\mathrm{d}\Phi_{12}}{\mathrm{d}t}, \tag{16}$$

where e is the induced electromotive force due the current in the power line. If we assume that

$$I_1(t) = I_0 \cos(2\pi f t)\,, \tag{17}$$

we get from (14) and (16) that

$$\begin{aligned} e &= -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu_0 I_0 \cos(2\pi f t) l}{2\pi}\left[\ln\left(\frac{b-z_1}{-z_1}\right) - \ln\left(\frac{b-z_2}{-z_2}\right)\right]\right) \\ &= \mu_0 I_0 f \sin(2\pi f t) l \left[\ln\left(\frac{b-z_1}{-z_1}\right) - \ln\left(\frac{b-z_2}{-z_2}\right)\right]. \end{aligned} \tag{18}$$

Thus, the amplitude e_0 of the induced electromotive force in the telephone line is given by

$$\underline{\underline{e_0 = \left|\mu_0 I_0 f l\left[\ln\left(\frac{b-z_1}{-z_1}\right) - \ln\left(\frac{b-z_2}{-z_2}\right)\right]\right|}}. \tag{19}$$

By inserting the values in (19), we get

$$\underline{\underline{e_0 = \left|4\pi\cdot 10^{-7}\cdot 100\cdot 50\cdot 500\left[\ln\left(\frac{10.3}{10.2}\right) - \ln\left(\frac{9.8}{9.7}\right)\right]\right|\mathrm{V} = 1.57\mathrm{mV}.}} \tag{20}$$

c) We can see from the figure that if we twirl the telephone line conductors, the magnetic flux within the closed loop C_2 will be close to zero: You get many small "loops" in series, where the surface norm for one loop is inverted compared to the adjacent loop. This causes the induced voltage in one loop to counteract the induced voltage in the next, so that the total induced voltage in C_2 is very small. The practical use for this is to eliminate the noise caused by induced electromotive forces on the telephone line.