# Prediction of stiffness of unbound granular materials based on the CBR value

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ABSTRACT: The unbound granular materials (UGM), base and subbase layers, play an essential role in the overall structural performance of thin pavement structures. They show complex stress dependent elasto-plastic behaviour under external loading. Therefore the UGM are commonly tested using the Repeated Load Triaxial (RLT) testing method to estimate the stiffness of the material by applying haversine loading pulses. The RLT testing method represents the actual stress situation quite adequately and gives satisfactorily estimates of the stiffness characteristics of UGM. However a RLT test is an elaborated test and rather expansive. A much simpler test that has been used for a long time in structural design of flexible pavements is the CBR (California Bearing Ratio) test. The CBR test is a simple and cheap test where the load-deformation curve is acquired while a plunger is penetrated into the material at a constant rate. In the literature one can find a number of relationships for UGM where the CBR value is used to predict the stiffness. To investigate if a connection between the two tests exists, twenty materials have been tested with both methods and the test results compared. The materials were of varying quality but all were fairly well graded. The results indicate that a simple power law can be used to predict the stiffness if the CBR-value is known.

KEY WORDS: Flexible pavement, granular materials, mechanical properties, stiffness, CBR values.

## 1 INTRODUCTION

New flexible pavement design methods are under development where the aim is to predict functional and structural conditions of roads over time. To do this the response of the pavement structure due to vehicle loading is calculated based on a mechanistic approach and thereafter a distress prediction (rutting, fatigue and thermal cracking) is carried out. The key parameters for the response analysis are the stress-strain relationships of the different layers of the pavement structure. Figure 1 illustrates the general stress regime experienced by an unbound base course element in a pavement structure as the result of a moving wheel load within the plane of the wheel track. Due to the wheel load, pulses of vertical and horizontal stress, accompanied by a double pulse of shear stress with a sign reversal, affect the element (Brown, 1996). A method, which represents this actual stress situation quite adequately, is the Repeated Load Triaxial (RLT) testing method, where the stiffness of the material is estimated by applying a number of haversine load pulses. However the RLT testing method is a rather time consuming and therefore expensive to use.

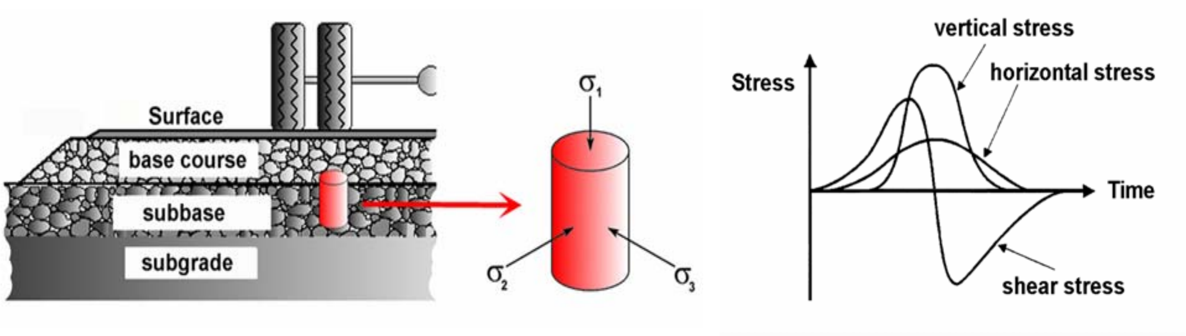


Figure 1: Stresses in a UGM layer. a) Typical pavement structure and stresses, b) induced stresses in a pavement element due to moving wheel load.

The CBR (California Bearing Ratio) value has been used for a long time in the structural design procedure of flexible pavements, even though it is known that it poorly represents the actual stress situation pavements experiences during traffic loading. One of the advantages of the CBR method is that it is a simple and cheap test method. In the CBR test a plunger is advanced once into a cylindrical mould of recompacted unbound granular material (UGM) at a constant rate. The load required to cause 2.54 and 5.08 mm insertion is recorded and expressed as a percentage of the load required for the same penetration in a certain standard material. This depth of insertion is more than sufficient to cause local failure in the material with large permanent deformations as a result which comprises many repeated light loading cycles.

In the literature one can find a number of relationships for UGM where the CBR value is used to predict the stiffness. To investigate if a connection exist between the CBR value and stiffness, twenty materials have been tested with both the RLT test as and CBR test equipment and the test results compared. The major advantages if a such relationship would exists is much cheaper and quicker estimation of the stiffness as the RLT is quite an elaborate test and rather expensive but the CBR test is a simple and cheap test.

## 2 STIFFNESS CHARACTERISTICS OF UGM

The stress regime in pavements due to vehicle loading can be expressed by introducing two stress invariants: the mean stress level $p$ and the deviatoric stress $q$, in which, for the case where $\sigma_2 = \sigma_3$, becomes

$$p = \frac{1}{3}\left(\sigma_1 + 2\sigma_3\right) \qquad \text{and} \qquad q = \sigma_1 - \sigma_3 \tag{1}$$

In a similar way strain invariants can be introduced. The volumetric strain $\varepsilon_v$ and deviatoric strain $\varepsilon_q$, are defined as

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \qquad \text{and} \qquad \varepsilon_q = \frac{2}{3}\left(\varepsilon_1 - \varepsilon_3\right) \tag{2}$$

in which $\varepsilon_1$ and $\varepsilon_3$ are the resilient axial strain and radial strain respectively, where it has been assumed in a comparable way as for the stresses that $\varepsilon_2 = \varepsilon_3$.

The stresses and strains are now interconnected through the material properties and can be expressed in a diagonal matrix:

$$\begin{bmatrix} \varepsilon_v \\ \varepsilon_q \end{bmatrix} = \frac{1}{M_r} \begin{bmatrix} 3(1-2\nu) & 0 \\ 0 & 2(1+\nu) \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{3}$$

where $M_r$ and $\nu$ are the material stiffness modulus (resilient modulus) and Poisson's ratio respectively, defined as:

$$M_r = \frac{q}{\varepsilon_1} \qquad \text{and} \qquad \nu = -\frac{\varepsilon_3}{\varepsilon_1} \tag{4}$$

It is well known that the stiffness modulus of UGM is stress dependent but the Poisson's ratio is not, or at least is to a much less extent and can usually be treated as a constant. A number of relationships exist to describe the stress dependency of the stiffness moduli. One of the most common and also one of the simplest is the so called $k$ - $\theta$ expression (Yoder and Witczak, 1975; Brown and Pappin, 1981; Correia et al., 1999; Schwartz, 2002):

$$M_r = k_1 \theta^{k_2} = k_1 \left( \frac{3p}{p_a} \right)^{k_2} \tag{5}$$

where $k_1$ and $k_2$ are experimentally determined constants and $p_a$ is a reference pressure, $p_a$ = 100 kPa.

By introducing equation (5) into (3) it is obvious that the stresses and the strains are interconnected in a nonlinear relationship, which can be written in matrix form as

$$[\boldsymbol{\varepsilon}] = [\mathbf{C}(p)][\boldsymbol{\sigma}] \tag{6}$$

in which $[\boldsymbol{\varepsilon}]^T = [\varepsilon_v, \varepsilon_q]^T$ and $[\boldsymbol{\sigma}]^T = [p, q]^T$ and $[\mathbf{C}(p)]$ is the compliance matrix.

To determine the stress strain relationship experimentally, and therefore the $k_1$, $k_2$ and $\nu$, a number of measurements is needed to cover the actual range of mean stresses $p$ and deviatoric stresses $q$ caused by different weights of the axle loads of the traffic. The RLT test method can be used for this estimation where different stress paths are applied (Correia et al., 1999; Erlingsson, 2000).

Many attempts have been made to connect values from CBR testing with stiffness modulus (Witczak et al., 1995; Hoff, 1999). In Table 1 four equations are shown where different organizations have made this attempt.

Table 1: Four equations, which have been used for the assessments of stiffness from results from CBR testing. Stiffness is in MPa, and CBR-values in %. (Witczak et al., 1995).

| Equation | Organization | Equation no. |
|---|---|---|
| $M_r = 10.35 \cdot CBR$ | Shell Oil | (7) |
| $M_r = 37.3 \cdot CBR^{0.711}$ | U.S. Army corps of Engineers (USAGE) | (8) |
| $M_r = 20.7 \cdot CBR^{0.65}$ | South African Council on Scientific and Industrial Research (CSIR) | (9) |
| $M_r = 17.25 \cdot CBR^{0.64}$ | Transport and Road Research Laboratory (TRRL) | (10) |

These four equations give very different results as one can see in Figure 2 where the predicted stiffness value according to the equations in Table 1 are given as function of the CBR value.

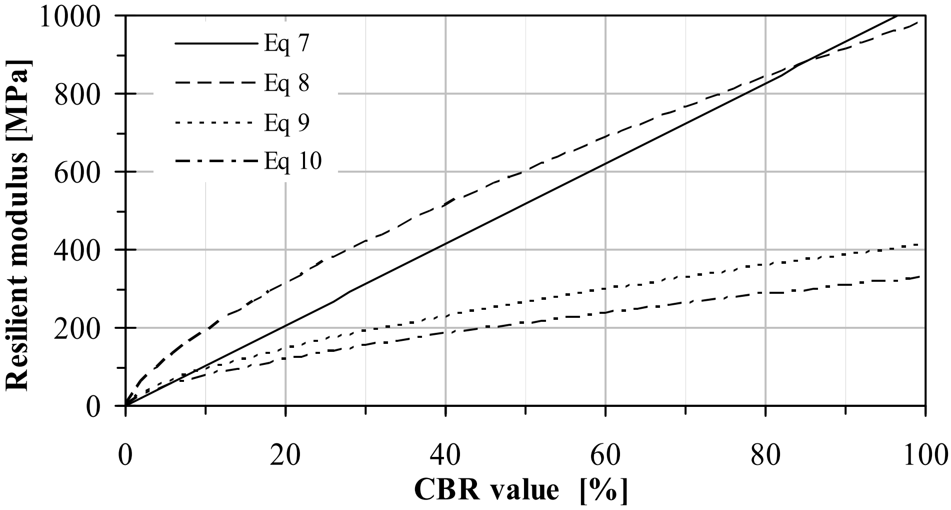


Figure 2: Predicted stiffness values as a function of CBR values according to the equations in Table 1.

## 3 TEST METHODS

### 3.1 CBR-testing

The CBR test is frequently used in the structural design procedure of flexible pavements. In the CBR test a specimen is compacted in a standard mould, with compaction corresponding to Standard Proctor compaction. The CBR value is obtained by measuring the relationship between force and penetration of a metal plunge into the material, at a given constant rate, see Figure 3. The load required to cause 2.54 and 5.08 mm insertion is recorded and expressed as a percentage of the load required for the same penetration in a certain standard material.

It is commonly accepted that the induced stresses involved during the CBR test poorly represents the actual stress situation that pavements experience during traffic loading. The depth of insertion is more than sufficient to cause local failure in the material with permanent deformation as a result, which comprises many repeated light loading cycles. Therefore it has been concluded that the CBR value is not suitable as an input parameter for mechanistic design methods. However as the CBR value is recorded as a stress at a given penetration (2.54 or 5.08 mm) one might argue that it represents the average slope of the stress deformation curve and should therefore represent the stiffness over that interval of deformation. For compacted materials with good grain size distribution curve were the aggregates are strong one would expect that the largest part of the deformation at 2.54 mm is due to elastic response of the material and only up to a small extent due to plastic deformation. Therefore the CBR-value might give some indications of the actual stiffness of the material.

In this project were the CBR test performed at four different moisture contents for each material, giving both the dry density as well as the CBR-value at these four moisture contents. The highest CBR value after 2.54 mm penetration is thereafter used in the comparison (Erlingsson and Magnusdottir, 2004).

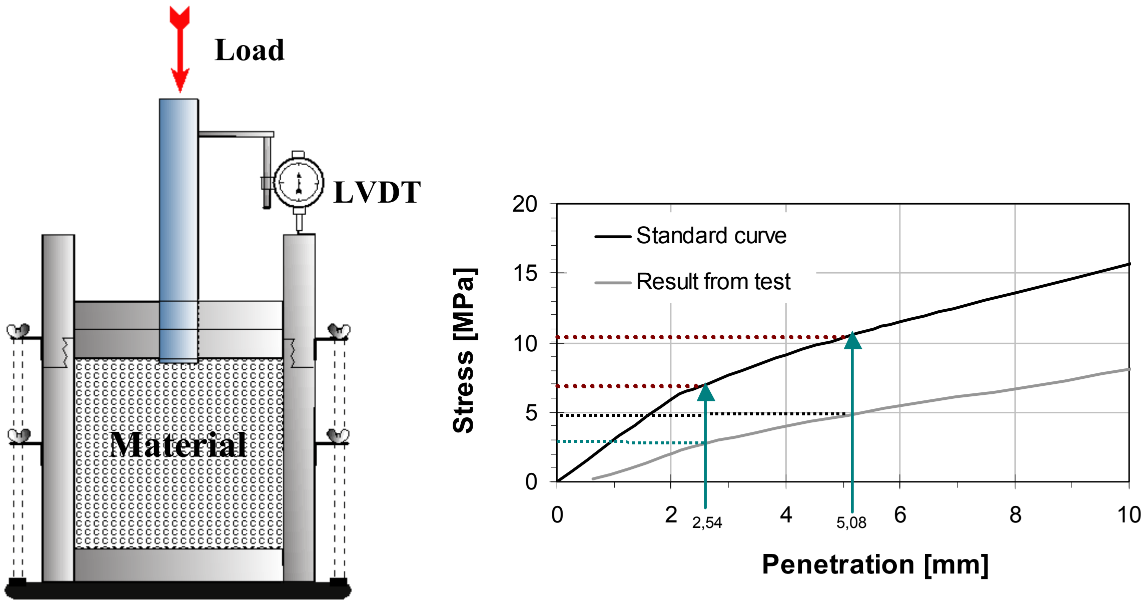


Figure 3: CBR test set-up and a schematic view of the evaluation of the CBR value.

## 3.2 Repeated Load Triaxial testing

In the RLT test performed here, 150 mm diameter specimen with a height of 300 mm were used. The materials were compacted according to the Proctor compaction method (Erlingsson and Magnusdottir, 2002).

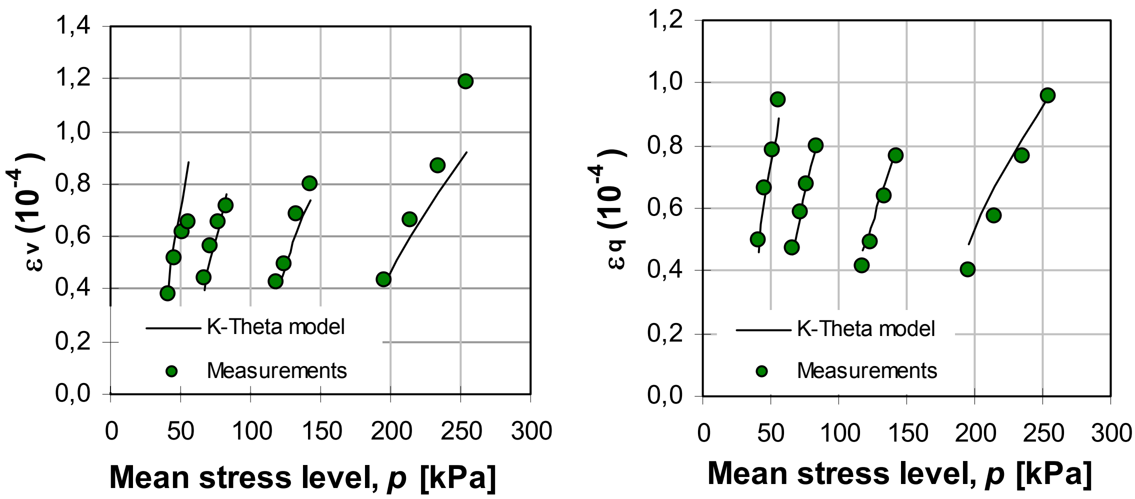


Figure 4: Typical results from RLT testing: comparison of measured and calculated strains. a) volumetric strain and b) deviatoric strain versus the mean applied stress level.

In the testing procedure a number of stress paths are applied and during each stress path 100 symmetric haversine load cycles are applied with a rise time of 50 ms (total length of pulse 0.1 sec) followed by a 0.9 sec rest time. During the last ten load cycle's data from the transducers as well as the axial load were collected to evaluate the specimen response. Typical results are shown in Figure 4 where the volumetric strain $\varepsilon_v$ and deviatoric strain $\varepsilon_q$ are plotted as functions of the mean stress level $p$. The data were further used to estimate $k_1$ and $k_2$ from equation (5) with the aid of the least square method, and these results are given as well in the figure.

## 4 THE MATERIALS

Totally twenty different granular materials were used in this study. They are all typical base and subbase materials of different qualities, and somewhat different petrology (mostly basalt). The maximum grain size of all the materials was 22.4 mm. Some parameters describing the materials can be found in Table 2.

Table 2: Percent fines, uniformity coefficient, coefficient of gradation, unified soil classification, Los Angeles abrasion value, flakiness index, specific gravity, dry density and the CBR value for the eighteen materials used in this research.

| Material | $D < 65 \mu m$ [%] | $C_u$ [-] | $C_c$ [-] | USCS | *LA* value [%] | *FI* [%] | $G_s$ [-] | *Density* $\rho_{dry}$ [kg/m$^3$] | *CBR* value [-] |
|---|---|---|---|---|---|---|---|---|---|
| **Bakkasel** | 3.4 | 7.90 | 2.3 | GW | 16.5 | 19.1 | 2.98 | 1974 | 52 |
| **Bjorgun** | 5.0 | 36.0 | 7.1 | GP-GM | 16.4 | 1.3 | 2.93 | 2102 | 91 |
| **Brjanslaekur** | 2.3 | 10.0 | 0.7 | GP | 17.1 | 9.8 | 2.96 | 2084 | 44 |
| **Glera** | 4.0 | 18.0 | 1.4 | GW | 17.3 | 12.3 | 2.76 | 1958 | 59 |
| **Haukadalsa** | 4.6 | 23.3 | 1.5 | GW | 19.2 | 7.4 | 2.95 | 2186 | 94 |
| **Haumelar** | 1.2 | 5.30 | 1.7 | GW | 17.6 | 8.4 | 2.91 | 1969 | 42 |
| **Holabru** | 3.4 | 40.9 | 3.9 | GP | 15.7 | 6.3 | 2.98 | 2305 | 140 |
| **Holmkelsa** | 2.9 | 14.0 | 1.4 | GW | 21.7 | 6.3 | 2.74 | 1967 | 68 |
| **Hraunaos** | 5.2 | 44.0 | 4.9 | GP-GM | 15.6 | 13.6 | 2.99 | 2262 | 119 |
| **Jökulsa a Dal** | 2.2 | 25.0 | 53.0 | GP | 17.6 | 8.6 | 2.93 | 2172 | 76 |
| **Jökulsa a Fjö.** | 2.4 | 13.3 | 2.3 | GW | 22.2 | 4.8 | 2.78 | 1010 | 49 |
| **Krossanes** | 1.2 | 7.5 | 1.6 | GW | 16.7 | 34.5 | 2.99 | 1896 | 30 |
| **Larkot** | 3.9 | 26.8 | 1.9 | GW | 16.1 | 10.5 | 2.89 | 2205 | 74 |
| **Laekjarbotnar** | 4.3 | 20.0 | 2.2 | GW | 30.0 | 3.0 | 2.75 | 1821 | 42 |
| **Markarfljot** | 1.4 | 23.8 | 1.3 | GW | 18.5 | 7.0 | 2.81 | 2032 | 81 |
| **Nordfjardara** | 1.8 | 10.8 | 1.6 | GW | 19.2 | 14.0 | 2.93 | 2031 | 42 |
| **Raudamelur** | 5.1 | 48.0 | 12.0 | GP-GM | 23.5 | 3.0 | 2.60 | 1873 | 89 |
| **Stora-Fells.** | 3.9 | 44.0 | 5.8 | GP | 25.5 | 10.2 | 2.90 | 2178 | 95 |
| **Vallholt** | 3.3 | 30.0 | 1.2 | GW | 17.4 | 8.2 | 2.87 | 2165 | 130 |
| **Vatnsskard** | 3.7 | 30.0 | 4.8 | GP | 36.8 | 2.8 | 2.61 | 1822 | 68 |

From Table 2 it can be seen that all the tested materials are rather low in fine contents, or *D* < 65 μm between 1.2 – 5.2 %. Twelve of the materials are classified as well graded gravel (GW) and only five as poorly graded gravel (GP) according to the Unified Soil Classification System (USCS). Further have fourteen of the materials *LA* abrasion values < 20 and would therefore be classified as material with good resistance to fragmentations according to CEN norm 1342. Thirteen of the materials have *FI* values lower than 10 and are therefore not considered flaky.

The grain size distribution curves of all the materials are given in Figure 5. Even though it is difficult to separate the different curves on the figure one can see that the materials have fairly good grain size distribution curves.

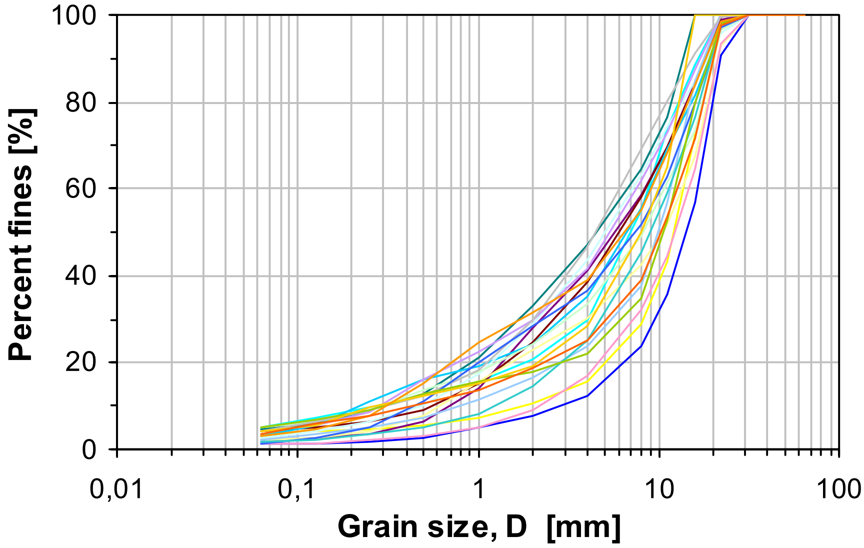


Figure 5: Overview of the grain size distribution of the twenty materials used in this study.

## 5 TEST RESULTS

Figure 6 shows the stiffness and the CBR values plotted as a function of the dry density of the materials. The stiffness and the CBR values shown are the values at moisture content which are approximately 2% below the optimum water content $w_{opt}$. There seems to be some relationship between CBR values and dry density of the materials. That is, materials with low dry density tend to have low CBR values and as the density increases the CBR-value of the material increases. For the resilient modulus this relationship seems to be much weaker if it exists at all.

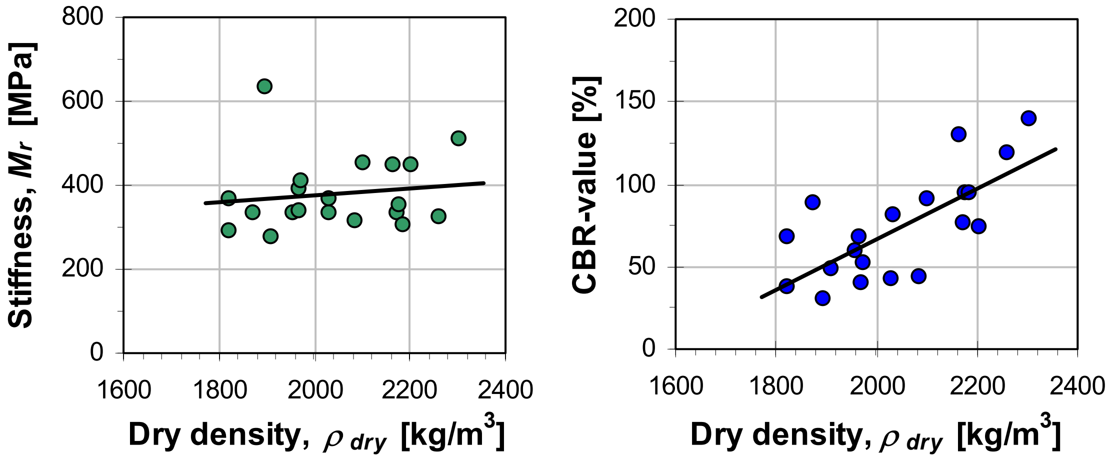


Figure 6: Measured maximum stiffness at mean stress level $p$ = 250 kPa and CBR value as a function of dry density for all the twenty materials.

To make a comparison between the resilient modulus and the CBR-value one must decide what stress level should be used as the resilient modulus is stress dependent. Here are mainly base course materials considered which are commonly used under thin surface dressings. Under such conditions it is not uncommon that induced vertical stress due to a 12 ton axle load with a tire pressure of 800 kPa is of the order 450 MPa at 10-15 cm depth under the surface and the horizontal stresses are approximately one third of that value (Erlingsson, 2002; Erlingsson & Ingason, 2004). The mean stress level $p$ becomes therefore approximately 250 kPa. This value is used here as a reference mean stress level. The predicted stiffness values based on the CBR values from Table 1 are given again in Figure 7 along with the actual stiffness measurements.

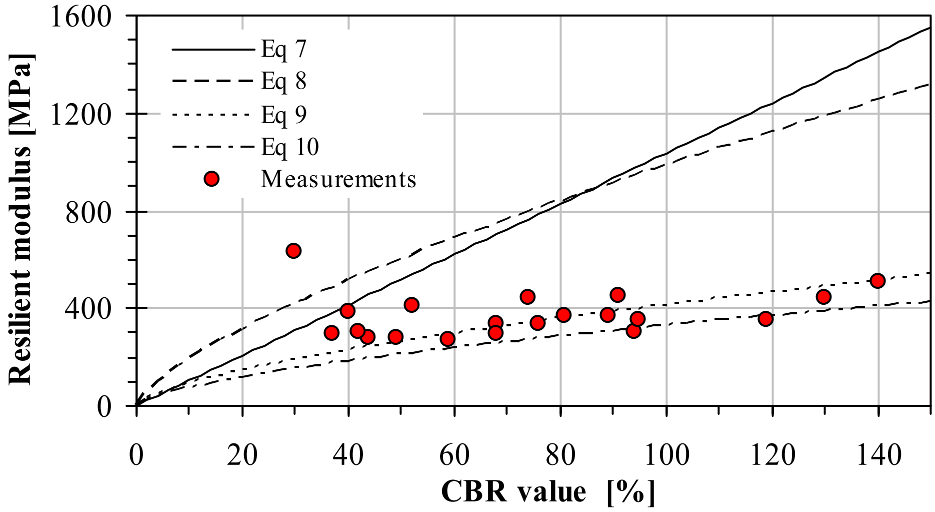


Figure 7: Predicted resilient modulus based on the CBR value from the four equations in Table 1 as well as the measured stiffness from the RLT test measurements at the mean stress level $p$ = 250 kPa.

One can see from Figure 7 that the last two equations in Table 1 seem to give a much better agreement with the measurements in particular for CBR values higher than approximately 60%. Here one must bear in mind that it has been decided to use the mean stress level as $p$ = 250 kPa when plotting the stiffness values. For higher stress levels it is possible that better agreement between the other two equations could be established.

Table 3 gives then the $k_1$ and $k_2$ values according to equation 5 for all the twenty materials. The values are based on the average value of testing at least three to four different test specimen of each material. One can see in Table 3 that the $k_1$ value ranges from 137 to 250 and that the $k_2$ value ranges from 0.272 to 0.630. Further it can be seen that the $k_2$ value lies for most of the materials in the range 0.33 – 0.45 and the average value is 0.37. If it is assumed that $k_2$ = 0.4 is sufficiently close to reality for the different materials one can back-calculate the $k_1$ value for the materials based on the prediction equations relating CBR value to the resilient modulus here using $p$ = 250 kPa, thus for equation 9 in Table 1 this gives:

$$\hat{k}_1 = \frac{M_r}{\left(\frac{3p}{p_a}\right)^{k_2}} = \frac{20.7 \cdot CBR^{0.65}}{\left(\frac{3 \cdot 250}{100}\right)^{0.4}} = 9.25 \cdot CBR^{0.65} \tag{11}$$

The last column in Table 3 gives the differences of the resilient modulus by using $k_1$ and $k_2$ in equation 5 and $\hat{k}_1$ and $k_2$ = 0.4 respectively at the mean stress level $p$ = 250 kPa. One can see for the materials that thirteen of the materials have CBR values higher than approximately 60% and of these nine materials give $\Delta$ lower that ±20% using $\hat{k}_1$ and $k_2$ = 0.4 in stead of $k_1$ and $k_2$ for predicting the stiffness. Two of the other four materials have rather high Flakiness Index, $FI$ = 13.6 and 10.5 respectively which might influence the result.

The Krossanes material shows the highest deviation of all the materials in the differences in the resilient modulus at the mean stress level $p$ = 250 kPa by the two methods. That material has a CBR value of 30 and is very flaky or $FI$ = 34.5 which is probably influencing the results.

Table 3: CBR value and the average of $k_1$ and $k_2$ values from the RLT testing, the calculated $\hat{k}_1$ value if $k_2 = 0.4$ and the % difference between the stiffness at $p$ = 250 kPa for all the eighteen materials.

| **Material** | **CBR-value [%]** | $k_1$ **[MPa]** | $k_2$ **[-]** | $\hat{k}_1$ **[MPa]** | $\Delta$ **[%]** |
|---|---|---|---|---|---|
| **Bakkasel** | 52 | 228.6 | 0.272 | 120.65 | 31.7 |
| **Björgun** | 91 | 152.3 | 0.630 | 173.59 | 24.0 |
| **Brjanslaekur** | 44 | 151.2 | 0.370 | 108.24 | 28.3 |
| **Glera** | 59 | 160.9 | 0.351 | 130.98 | 10.2 |
| **Haukadalsa** | 94 | 151.7 | 0.344 | 177.29 | -30.8 |
| **Haumelar** | 40 | 175.5 | 0.394 | 101.74 | 41.3 |
| **Holabru** | 140 | 214.6 | 0.430 | 229.68 | -0.7 |
| **Holmkelsa** | 68 | 159.6 | 0.374 | 143.64 | 5.2 |
| **Hraunaos** | 119 | 141.9 | 0.400 | 206.66 | -45.6 |
| **Jökulsa a Dal** | 76 | 137.7 | 0.442 | 154.41 | -3.0 |
| **Jökulsa a Fjöllum** | 49 | 151.6 | 0.296 | 116.08 | 5.6 |
| **Krossanes** | 30 | 464.2 | 0.153 | 84.39 | 70.1 |
| **Larkot** | 74 | 249.4 | 0.288 | 151.75 | 23.7 |
| **Laekjarbotnar** | 37 | 159.7 | 0.414 | 96.71 | 41.1 |
| **Markarfljot** | 81 | 157.1 | 0.419 | 160.94 | 1.4 |
| **Nordfjardara** | 42 | 153.3 | 0.383 | 105.02 | 29.1 |
| **Raudamelur** | 89 | 164.7 | 0.351 | 171.10 | -14.7 |
| **Stora-Fellsöxl** | 95 | 177.9 | 0.339 | 178.51 | -13.5 |
| **Vallholt** | 130 | 176.0 | 0.462 | 218.88 | -9.8 |
| **Vatnsskard** | 68 | 144.9 | 0.340 | 143.64 | -11.9 |

The resilient modulus versus mean stress level for two of the materials are given in Figure 8 where both the actual measurements using $k_1$ and $k_2$ are given as well as the predicted stiffness based on $\hat{k}_1$ and $k_2 = 0.4$.

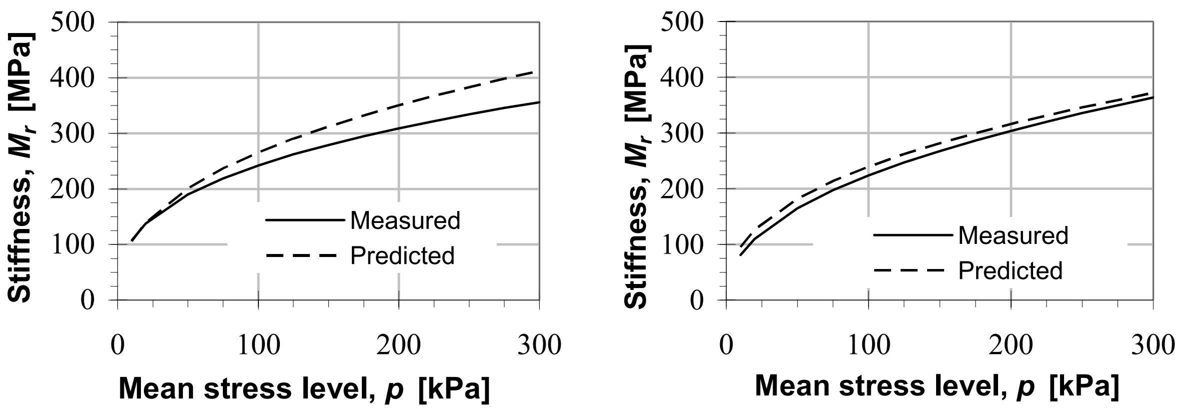


Figure 8: Resilient modulus versus mean stress level for two materials based on actual $k_1$ and $k_2$ values and $\hat{k}_1$ and $k_2 = 0.4$ respectively. a) the Raudamelur material with $\Delta$ = -14.7% and b) the Jökulsa a Dal material with $\Delta$ = -3.0%.

As one can see the difference is not very significant. For the Raudamelur material the predicted curve deviates from the measurements as the mean stress level increases. This is due to that the actual $k_2$ value 0.351 is lower that 0.4 which is used to create the predicted curve. For the other material, Jökulsa a Dal, the $k_2$ value 0.442 is higher that 0.4 so the two curves get closer as the mean stress level increases.

## 6 CONCLUSIONS

Here twenty unbound granular base course and subbase material have been tested with a CBR test to obtain the CBR value and RLT test to estimate their resilient modulus. All the materials have fairly good grain size distribution curves with low fines contents and twelve of the materials are classified as well graded gravel according to the USCS.

Based on the test results it is concluded that the CBR value of the unbound granular materials show some dependency of the dry unit weight of the material. This is true at least for well graded gravel with a low fines content and moisture content values close to $w_{opt} - 2$ %. It is further concluded that for materials with higher CBR values than approximately 60%, a simple relationship can be used to estimate their resilient modulus based on the CBR value. For a mean stress level $p$ = 250 kPa the prediction equation from the South African Council on Scientific and Industrial Research (equation 9) gives the best results. It is further concluded that if the $k$ - $\theta$ expression is used for describing the stress dependency of the stiffness modulus a value of $k_2 = 0.4$ and then estimating the $k_1$ value based on the CBR value seems to give a quite satisfactory estimation of the stiffness value.

## 7 ACKNOVLEDGEMENTS

The Public Roads Administration in Iceland has sponsored the work described in the paper.

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