

Evaluation of grain shape characterization methods for unbound aggregates

L. Uthus

Norwegian University of Science and Technology, Department of Civil and Transport Engineering, Trondheim, Norway

I. Hoff

SINTEF Technology and Society, Road and Railway engineering, Trondheim, Norway

I. Horvli

Norwegian University of Science and Technology, Department of Civil and Transport Engineering, Trondheim, Norway

ABSTRACT: A laboratory study is performed on an unbound aggregate of gneiss commonly used as base course in road pavements. Aggregate samples with four different grain shapes from the same quarry have been tested. The objective of this study was to evaluate different methods of grain shape characterization to see which methods that gives the most reliable results for the same materials on the parameters describing shape, angularity and surface texture.

One group of methods for grain shape characterization is simple petrologic methods in combination with physical measurements of the aggregate shape. These methods are based on visual classifications of rock type and measurement of the aggregate dimensions giving several shape characteristics as flatness and elongation ratio, angularity, degree of sphericity, flakiness index, shape index etc. The most common test in this group is the flakiness index test.

Image analyses are more sophisticated in their characterization of the aggregate shape. These methods are also more powerful in their ability to characterize surface roughness. Two dimensional analysis give a picture of each grain in only two dimensions, i.e. the projection of the grain on one plane in a random position. The most advanced methods are the three dimensional image analysis.

KEY WORDS: Grain shape characterization, aggregate shape, image analysis

1. INTRODUCTION

The geometrical irregularities of an aggregate particle are of great importance for the behaviour of the aggregates. Aggregate particles appear in different shapes depending on its origin and later processes. Aggregate particles worn by wind, water or glaciers appear rounded while particles from crushed rock appear angular (Wadell 1932).

The grain shape of aggregates influences the material gradation obtained by sieving. According to Lees (1964) flaky particles tends to pass diagonally through the sieves with square holes. The shape also has a significant influence on the volume of the particles retained on each sieve. As shown by Lees (1964) the rod-shaped particles are about 2.5 times the volume of the disc shaped particles.

Requirements in design guidelines for grain shape give limitations for flaky and elongated particles; hence promoting equidimensional or cubic aggregates. Parameters as roundness, angularity and surface texture are only given in the guidelines as a requirement for percentage of crushed and broken surfaces. More sophisticated methods as image analysis are not yet practical or accepted for everyday engineering work, but they have been used by researchers.

There are many methods for quantification of the grain shape; from the simple visual methods and physical measurements to complex three dimensional image analysis. The manual methods are laborious and tedious measurements, and the more sophisticated methods are expensive but more effective and easier to operate. Some of the methods available have been evaluated with respect to quality of output.

2. METHODS FOR QUANTIFICATION OF AGGREGATE GRAIN SHAPE

2.1 Visual methods and sieving

Wadell (1932) questioned the use of measurements to quantify the shape of aggregate particles: *How is it possible to define the breadth and thickness of an irregular particle?* These types of characterizations will always be estimates and approximations. However, they are easy to perform and widely used.

2.1.1 Description of particle shape, angularity and surface texture

The grain shape is one of many important parameters influencing the response of unbound aggregates. Lees (1964) Barksdale and Itani (1989) categorized the grains in 4 different shapes; disc, equidimensional, blade or rod shaped. Janoo (1998) organized the description of the different shapes in a table.

Table 1: Description of aggregate shape

Aggregate shape	Description
Disc	Slabby in appearance, but not elongated
Equidimensional	Neither slabby appearance nor elongated
Blade	Slabby appearance
Rod	Elongation, but not slabby in appearance

Lees (1964) found that these 4 broad categories permitted quite a large range of particle shape characteristics within each classification. He concluded that this classification was not suitable for research purposes and that it would probably be better to classify the particle shape by the flatness and the elongation ratio.

The flatness ratio (p) is the ratio of the short length (thickness) to the intermediate length (width). The elongation ratio (q) is the ratio of the intermediate length to the longest length (length). By combining the flatness- and the elongation ratio, the shape of the aggregates can be described by a shape factor (F) and the sphericity (ψ). The shape factor is the ratio of the elongation ratio and the flatness ratio (Eq. 1).

$$F = \frac{p}{q} \quad (\text{Eq. 1})$$

A round or cubical particle will have a shape factor equal to 1. If the shape factor is less than 1, the particle is more elongated and thin. A blade shaped particle will have a shape factor greater than 1.

The sphericity is defined as the ratio of the surface area of a sphere having the same volume as the aggregate particle. The sphericity can also be expressed by the flatness- and elongation ratios as shown in Equation 2. The sphericity varies from values near 0 to values near 1.0 for perfect spheres.

$$\psi = \frac{12.8(\sqrt[3]{p^2q})}{1 + p(1 + q) + 6\sqrt{1 + p^2(1 + q^2)}} \quad (\text{Eq. 2})$$

The flatness- and elongation ratio and the shape factor and the sphericity are combined in the diagram below. In the diagram the aggregates can be classified as disc, equidimensional, blade- and rod shaped as described in Table 1. Equidimensional means either cubic or round particles.

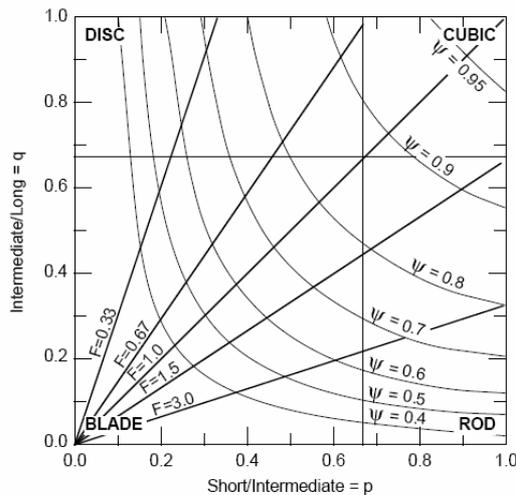


Figure1: Aggregate classification chart (Lees, 1964, Janoo, 1998)

2.1.2 Norwegian classification of aggregates for concrete

The aim of this testing procedure is to determine the percentage of flaky and elongated grains in a sample (Norwegian Control Board for Concrete Products, 2001). For use in concrete the grain shape of the material is of importance to the workability of the concrete mass and also the amount of water needed.

The procedure gives 3 different groups of grain shape: cubic, flaky and elongated in combination with rounded/subrounded and angular/subangular. For measuring the grains a slide calliper is used in combination with visual characterization.

Flaky and elongated grains are determined by the relation between the length, width and thickness of the grain. The length is defined as the longest axis of the grain. The width is the second longest axis of the grain and the thickness is the shortest axis.

A grain is classified as **flaky** if: $\frac{\text{width}}{\text{thickness}} > 2.0$

A grain is classified as **elongated** if: $\frac{\text{length}}{\text{thickness}} > 2.5$

A grain is classified as **cubic** if it is not flaky or elongated, that means:

$$\frac{\text{width}}{\text{thickness}} < 2.0 \text{ and } : \frac{\text{length}}{\text{thickness}} < 2.5$$

The cases of doubt are then measured by a slide calliper to determine the grain shape.

2.1.3 Shape Index

The shape index (EN 933-4:1999) is a method for determining the elongation of coarse aggregate grains. The test is performed on aggregate with grain size ranging from 4 mm and up to 63 mm. Particle length is described as the maximum dimension of a particle as defined by the greatest distance apart of two parallel planes tangential to the particle surface. Particle thickness is described as the minimum dimension of a particle as defined by the least distance apart of two parallel planes tangential to the particle surface

A special made particle slide gauge is used for determination of the category for each aggregate grain. This is made with the use of two openings or slots, one to measure the length and another opening that is 1/3 of the opening for the length. In this way it is easy to find if the length is more or less than 3 times the thickness. According to the result for each particle the sample is divided into the two categories; cubical and non cubical, where the non cubical is the particles where the thickness is less than 1/3 of the length. The shape index is calculated as the ratio between the mass of non-cubical particles and the total mass of particles tested

2.1.4 Flakiness Index

This method (EN-933-3:1997) is commonly used in the European countries with requirements for aggregates for several material types in road pavements. The test consists of two sieving operations. During the first operation test sieves are used to separate the sample into various particle size fractions. Each of the fractions are then sieved using bar sieves.

The flakiness index can be calculated for each fraction within the sample and for the whole sample. The overall index is calculated as the total mass of particles passing the bar sieves expressed as a percentage of the total dry mass of the whole sample.

2.2 Image analysis

Digital image processing for characterization of aggregate shape has been used for research purposes in many years. Several researchers have used both 2D and 3D image analysis methods to quantify the grain shape parameter.

Digital image processing is a technique of using pictures on a digital form and applying various mathematical procedures to get digital information from the image.

2.2.1 Two dimensional quantification of grain shape by the QMOT image analysis system

Image analysis is an advanced way to evaluate and quantify the shape of an aggregate. In the simplest form this is to arrange grains on a white background and take pictures. Then the pictures show a two dimensional projection of the aggregate grain shape.

By using a computer with a suitable programme for processing the pictures, it is possible to calculate almost any geometrical parameter in two dimensions. The Quebec Ministry of Transportation (QMOT) uses image analysis on a regular basis to verify the angularity of its hot-mix aggregates (Janoo 1998).

From the two dimensional image analysis two different parameters are found; the roundness and the roughness of the aggregates. An image analysis programme is used to track the two dimensional projection of the aggregates and get rid of shades and other disturbance.

For the image analysis the following basic measurements are made:

- 1) Area
- 2) Perimeter

- 3) Ferets
- 4) x-y coordinates

The area is calculated as the number of pixels of an object that form an object and the perimeter is determined by counting the number of pixels touch the background or the projection of the object. This parameter will depend on the resolution of the image, it is more sensitive to variations in texture at high resolutions and to changes in angularity at relatively low resolution.

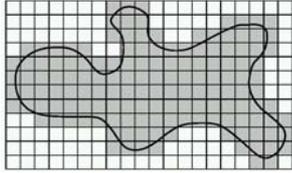


Figure 2a: Area of an aggregate (Janoo 1998)

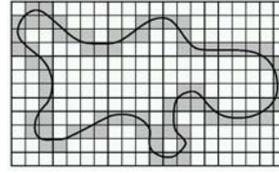


Figure 2b Perimeter of an aggregate (Janoo 1998)

Ferets are straight-line measurements made between two tangents as shown in figure 3a. From these measurements it is possible to obtain expressions for both the roundness and the roughness of an aggregate.

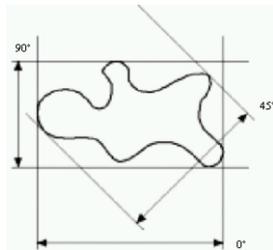


Figure 3a: Feret measurement (Janoo 1998)

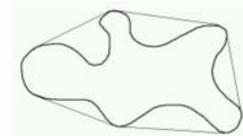


Figure 3b: Illustration of convex perimeter (Janoo 1998)

The roundness/angularity index (Janoo 1998) is calculated by the following formula:

$$R_n = \frac{4\pi A}{p^2} \quad (\text{Eq. 3})$$

where,

R_n = Roundness index

A = area of aggregate

p = perimeter of aggregate

This parameter typically has a value less than 100. A perfectly circular object will have a value of 100, but because of the square pixel simplification the best that can be obtained is 95. An object with irregular surface will have a smaller value.

The roughness of the grains is calculated by the following formula (Janoo 1998):

$$r = \frac{p}{p_c} \quad (\text{Eq. 4})$$

where,

r = roughness

p = perimeter of aggregate

p_c = convex perimeter of aggregate

The roughness of an aggregate grain is defined by the ratio of perimeter to the convex perimeter. For a smooth material the roughness factor equals 1.00, the roughness factor increases by the increasing roughness of a particle.

2.2.2 Three dimensional quantification of grain shape by the UI-AIA system

An aggregate particle has three dimensions in nature, so a two dimensional projection will not tell the whole truth. The method presented is one of the sophisticated and advanced methods available. This type of equipment demands a lot of money and effort to build up, but over the years many methods have been developed with different approaches.

An image analyzer has been developed at the University of Illinois to provide a reliable means of automating the determination of coarse aggregate size and shape properties. Flat and elongated ratio, gradation and angularity are the factors investigated in a fast and efficient way.

The apparatus uses three cameras to collect aggregate images from three orthogonal directions. The particles that are analyzed are fed on to a conveyor belt system, which carries the particles – one by one – towards the orthogonally placed cameras. Each particle will be detected by the cameras as they travel into the field of view of a sensor. The cameras are synchronized so the three pictures are taken at the exact same time.

The angularity index described by Rao et al. (2002) was developed as a part of the UI-AIA analysis package. The basis theory has been carefully verified with typical coarse aggregate shapes in order to develop an objective tool to characterize aggregate grain shape.

The image analysis procedure first provides an angularity value for each of the three 2D images collected from the three views. Then the angularity is established as a weighed average of all three views.

$$AI_{\text{Particle}} = \frac{\text{Ang.}(\text{front}) \cdot \text{Area}(\text{front}) + \text{Ang.}(\text{top}) \cdot \text{Area}(\text{top}) + \text{Ang.}(\text{side}) \cdot \text{Area}(\text{side})}{\text{Area}(\text{front}) + \text{Area}(\text{top}) + \text{Area}(\text{side})} \quad (\text{Eq. 5})$$

where;

AI_{Particle} = Angularity index for the particle

Ang. = Angularity for one view

Area = Area for one view

The AI for a sample is the average of the AI for all grains weighed by weight. The unit for AI is degrees.

The UI-AIA system can also be used to determine the surface texture of coarse aggregates by using an erosion-dilation technique (Rao et al. 2003). This is a well known technique in image analysis where the erosion operation is a procedure to obtain the first imaging morphologic parameter. Masad et al. (1999) used the erosion-dilation technique to determine the surface texture of fine aggregates.

Erosion is a morphologic operation by which all boundary pixels are removed from a binary picture, leaving the object one pixel less dense along the boundary. Dilation is the reverse of erosion, where a layer of pixels is added to the object boundary to create a simplified form of the original object. The area lost under the erosion-dilation process as a percentage of the total area of the original image is denoted by the surface parameter ST:

$$ST = \frac{A_1 - A_2}{A_1} \cdot 100 \quad (\text{Eq. 6})$$

where;

A_1 = Area (in pixels) of the two dimensional projection before erosion-dilation

A_2 = Area of object after erosion-dilation

The surface texture ST is calculated as the average of the surface texture values for each view weighed by their individual areas as shown in Equation 7.

$$ST_{\text{particle}} = \frac{ST(\text{front}) \cdot \text{Area}(\text{front}) + ST(\text{top}) \cdot \text{Area}(\text{top}) + ST(\text{side}) \cdot \text{Area}(\text{side})}{\text{Area}(\text{front}) + \text{Area}(\text{top}) + \text{Area}(\text{side})} \quad (\text{Eq. 7})$$

where;

ST_{particle} = Surface texture for the particle

ST = Surface texture for one view

Area = Area for one view

3. RESULTS

3.1 Visual methods and sieving

The characterization of grain shape according to Lees (1964), and Barksdale and Itani (1989) is used to characterize aggregate from Askøy with 4 different aggregate shapes. Table 4, Table 5 and Figure 4 shows the results from the Lees method for the different grain shapes. As can be seen both from Table 4 and Figure 2 the results are not giving any clear distinction between the series. However, from a statistical point of view the results are significant when testing in a 95% confidence interval. Only the two values for the form factor, which is almost similar, are not significant.

Table 4: The flatness- and elongation ratio of the series (Lees, 1964)

Material type / sample	Aggregate shape (analysis result)					
	Flatness Ratio (p)			Elongation Ratio (q)		
	Mean	Variance	Std. dev.	Mean	Variance	Std. dev.
Cubic	0.65	0.00017	0.013	0.74	0.00011	0.011
Cubic Rounded	0.69	0.00016	0.013	0.78	0.000096	0.0098
Flaky	0.52	0.00026	0.016	0.68	0.00016	0.013
Flaky Rounded	0.63	0.00017	0.013	0.62	0.00014	0.012

Table 5: The shape factor and sphericity of the series

Material type / sample	Aggregate shape (analysis result)					
	Shape factor			Sphericity		
	Mean	Variance	Std. dev.	Mean	Variance	Std. dev.
Cubic	0.92	0.00070	0.026	0.86	0.000021	0.0046
Cubic Rounded	0.92	0.00087	0.029	0.88	0.000016	0.0039
Flaky	0.83	0.0013	0.036	0.77	0.000075	0.0086
Flaky Rounded	1.08	0.0014	0.037	0.82	0.000034	0.0059

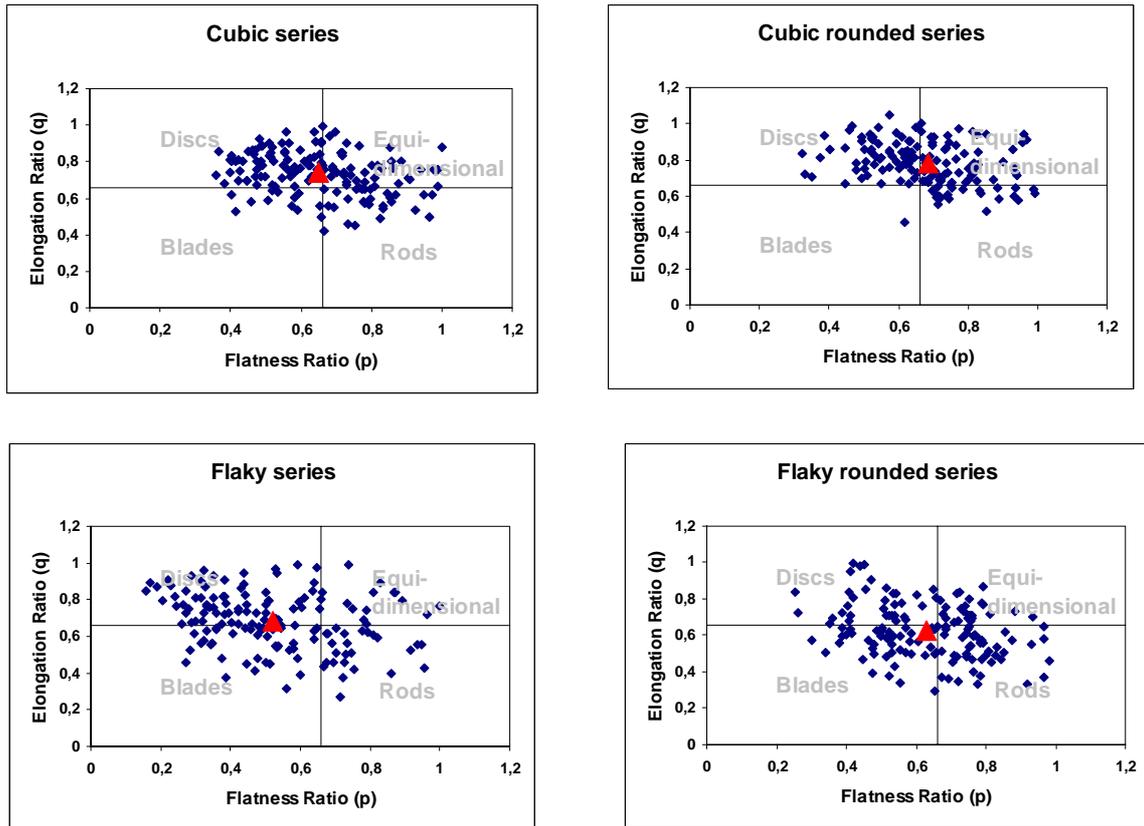


Figure 4: Results from characterization of grain shape after Lees (1964) method

The main part of the cubic series fall in the category of disc shape, while cubic rounded series tends more to be equidimensional. The flaky series falls in between the categories of disc and blade shape while the flaky rounded material tends to be more blade shaped. These results are expected results from the known differences in the grain shapes. The shape factor and sphericity in Table 5 does not give any clear difference between the series.

3.1.3 The Norwegian method for classification of aggregates for concrete

The Norwegian method for classification of aggregates for concrete is a visual method, so it will not give results to analyze statistically. As can be seen from Table 6 the cubic material tends to be cubic angular (69 %), while a smaller portion is flaky/elongated and angular (26 %). The cubic rounded material tends to be cubic rounded/subrounded (75%) while some grains are flaky/elongated and rounded/subrounded (25 %). For the flaky series most of the grains falls under the category of flaky/elongated and angular (60 %), but some grains are cubic angular (36 %). A large portion of the flaky rounded grains are flaky/elongated and rounded/subrounded (61 %) and some are cubic angular (39 %).

Table 6: Results from the Norwegian method for classification of aggregates for concrete

Material type / sample	Aggregate shape (analysis result)			
	Cubic Rounded/subrounded	Cubic Angular	Flaky/elongated Rounded/subrounded	Flaky/elongated Angular
Cubic	2	69	3	26
Cubic Rounded	75	0	25	0
Flaky	3	36	1	60
Flaky Rounded	39	0	61	0

3.1.4 Shape Index and Flakiness Index

The shape and flakiness indexes according to the EN standard (EN-933-3:1997 and EN 933-4:1999) seems to distinguish relatively well between the materials, see Table 7. Here we observe considerably lower shape index for the cubic rounded material than for the cubic, and considerably higher shape index for the flaky rounded than for the flaky. The same tendency is shown also for the flakiness index. The shape indexes are three orders of magnitude lower for the cubic and cubic rounded materials than for the flaky and flaky rounded. The flakiness indexes are also relative lower for the cubic variants than for the flaky, but this parameter does not show such big a difference. In this case is it impossible to use a statistical approach, as these tests are bulk tests, and only two parallels are tested.

Table 7: Results from the shape index and flakiness index

Material type / sample	Aggregate shape (analysis result)	
	Shape Index	Flakiness Index
Cubic	0.083	10.99
Cubic rounded	0.056	8.08
Flaky	55.5	12.42
Flaky rounded	63.3	20.34

3.2 Image analysis

Results from the applied two dimensional image method frequently used by The Quebec Ministry of Transportation are shown in Table 8. The results are as expected both for the roundness and the roughness. The roundness results shows that the cubic rounded series is the one with highest roundness and the flaky series is the most angular. The flaky rounded series have a lower roundness than the cubic series. This probably has to do with the elongation of these grains which gives a high perimeter value. The results of the roughness value shows that both the rounded series has a lower roughness than the more angular series, with the flaky series giving the highest roughness value.

The differences in the roundness and roughness parameters are relatively small compared to the accuracy of the method, however the results are significantly different when testing for significance for a 95 % confidence interval. The method should distinguish better between such different grain shapes.

Table 8: Results from the two dimensional image analysis

Material type / sample	Aggregate shape (analyse result)					
	Roundness			Roughness		
	Mean	Variance	Std. dev	Mean	Variance	Std. dev.
Cubic	76	43	6.5	1.048	0.00011	0.01
Cubic rounded	80	31	5.6	1.037	0.00042	0.02
Flaky	67	140	11.8	1.10	0.035	0.2
Flaky rounded	74	47	6.9	1.044	0.0000070	0.003

3.2.2 Three dimensional image analysis (UI-AIA analysis)

Table 8: Results from the three dimensional image analysis

Material type / sample	Aggregate shape (analyse result)					
	Angularity Index (AI)			Surface Texture Index (ST)		
	Mean	Variance	Std. dev.	Mean	Variance	Std. dev.
Cubic	369.9	82.5	9.1	0.83	0.0011	0.033
Cubic Rounded	270.0	86.5	9.3	0.54	0.0042	0.065
Flaky	417.1	86.8	9.3	1.06	0.0023	0.048
Flaky Rounded	317.2	129.38	11.4	0.79	0.0043	0.066

A three dimensional image analyse developed on the University of Illinois (Rao et al, 2001, 2002) is used to analyse the material from Askøy. The angularity index as well as the surface texture index is significantly different for all the material types; Cubic, cubic rounded, flaky, flaky rounded according to this method when testing for a 95 % confidence interval. The method is therefore capable to distinguish very well between all material types in this study

4. DISCUSSION

In this study many different methods has been applied on four different grain shapes of the same material. It is difficult to say something about the quality of the methods because of the big differences in the way they are treating the data.

- Lees method

As can be seen from the results the method does not clearly separate the different series. Statistically this method is also not a good method as some of the values are not significantly different for the different series. This method is based on physical measurements of the length, breadth and width, which according to Wadell (1932) is difficult when working with irregular particles.

- The Norwegian method for classification of aggregates for concrete

This method is subjective, which means that the results are to some degree dependent on the operator. It is therefore important to use the same operator for all the series to ensure that the results can be compared. As shown in the results this method clearly separated the different series. However the categories are not detailed, so it will not distinguish well between series that is almost similar in grain shape.

- Flakiness and Shape Index

The results show that the flakiness index separates the series to some extent, but there were expected to be a larger difference between the flaky and the cubic series. An explanation could

be that the flaky series are not that flaky, only elongated. The results from the shape index shows a big difference between the flaky and the cubic series, which was expected. These tests distinguishes well between materials with i.e. typically flaky or typically elongated grain shapes, but will not separate material with minor differences in grain shape.

- Two dimensional image analysis

Janoo (1998) found that the two dimensional image analysis (QMOT image analysis system) did not distinguish well between crushed and rounded aggregates regarding roundness and angularity. It must be noted that this study was done on a very limited range of materials with no additional material performance testing.

In this study the same conclusions may be drawn. The method separates the series with significant differences, but does not give a clear distinction between the materials which can easily be separated visually. One of the disadvantages of the method is that you only measure the two dimensional projection of the aggregates is measured. As aggregates tend to lay with the flat side down, this will give some problems to the quality of the output. The elongation is always measured, but a proper measure for the flatness or the combination of flatness and elongation will be more difficult.

The resolution of the images is of great importance in image analysis. As found by other researchers, the resolution is of importance to the quality of the output depending on the parameter investigated. The lower resolution, the less accurate the analysis will be.

- Three dimensional image analysis

The results clearly show that this method is the best method for characterizing this series. For the two parameters chosen (angularity index and surface texture index) the results are as expected and correspond to what can be observed visually.

A disadvantage of the UI-AIA method is the fact that it is not able to analyze dark colored aggregates. This is because of the black colour of the conveyor belt transporting the aggregates. This problem could be avoided by using a light conveyor belt. It is also a method that demands expensive equipment.

4. CONCLUSIONS

Simple methods of grain shape characterization distinguish well between aggregates with a clear difference in grain shape, but do not separate materials which has almost similar grain shape. A combination between several methods, e.g. flakiness and shape index, may give the information needed on grain shape.

The two dimensional image analysis (QMOT image analysis system) performed on the processed aggregates did not distinguishing good enough between crushed and rounded aggregates.

As can be seen from the results the three dimensional image analysis is the best method for good quality output. There may also be some resistance from the industry to use new tests that do not give the same results as traditional tests, even though the image analysis is more accurate. The question is how detailed information is needed to separate materials, and how the differences found would affect the behaviour of the material in a base layer.

REFERENCES

- American Society for Testing and Materials (ASTM), 2000. *Standard test method for index of aggregate particle shape and texture*, ASTM Designation D 3398-00, Philadelphia.
- American Society for Testing and Materials (ASTM), 2000. *Standard practice for description and identification of soils (Visual-Manual Procedure)*, ASTM Designation D 2488-00, Philadelphia
- Barksdale, R and Itani, S, 1989. *Influence of Aggregate shape on Base Behavior*, Paper Transportation Research Record 1227, Transportation Research Board, National Research Council, Washington DC, pp 173-181
- Barksdale, R, Kemp, M.A., Sheffield, W. J. and Hubbard, J. L., 1991. *Measurement of Aggregate Shape, Surface Area and Roughness*, Paper Transportation Research Record 1301, Transportation Research Board, National Research Council, Washington DC, pp 107-116.
- European Committee for Standardization, 1997. *Tests for geometrical properties of aggregates. Part 3: Determination of particle shape, Flakiness Index*. EN-933-3:1997.
- European Committee for Standardization, 1999. *Tests for geometrical properties of aggregates. Part 4: Determination of particle shape, Shape Index*. EN 933-4:1999.
- Janoo, V, 1998. *Quantification of Shape, Angularity, and Surface Texture of Base Course Materials*. Special Report 98-1, US Army Corps of Engineers, Cold Regions Research & Engineering Laboratory
- Janoo, V and Bayer II, John J, 2001. *The Effect of Aggregate Angularity on Base Course Performance*. Technical Report TR-01-14, US Army Corps of Engineers, Cold Regions Research & Engineering Laboratory
- Lees, G., 1964. *The measurement of particle shape and its influence in engineering materials*. Journal of the British Granite and Whinestone Federation, London, pp 1-22
- Maerz, N. H, 1998. *Aggregate Sizing and Shape Determination Using Digital Image Processing*. Center For Aggregate Research (ICAR) Sixth Annual Symposium Proceedings, St Louis Missouri, pp 195-203.
- Masad, E. and Button, J. W, 2000. *Unified Imaging Approach for Measuring Aggregate Angularity*. The International Journal of Computer-Aided Civil and Infrastructure Engineering. Vol 15 No 4. pp 273-280
- Norwegian Control Board for Concrete Products, 2001. *Metoder for proving av betongtilslag. Klasse P Betongtilslag (In Norwegian)*.
- Rao, C., Tutumluer, E. and Stefanski, J.A., 2001. *Coarse Aggregate Shape and Size Properties Using a New Image Analyzer*. ASTM Journal of Testing and Evaluation, Vol. 29, No. 5, pp 79-89
- Rao, C., Tutumluer, E. and Kim, I. T., 2002. *Quantification of Coarse Aggregate Angularity Based on Image Analysis*. Paper Transportation Research Record 1787, Transportation Research Board, National Research Council, Washington DC.
- Rao, C., Pan, T. and Tutumluer, E, 2003. *Determination of Coarse Aggregate Surface Texture Using Image Analysis*. Paper for the 16th ASCE Engineering Mechanics Conference, University of Washington, Seattle.
- Rex, H. M. and Peck, R. A., 1956. *A Laboratory Test to Evaluate the Shape and Surface Texture of Fine Aggregate particles*. Public Roads 29, pp 118-120
- Wadell, H., 1932. *Volume, shape and roundness of rock particles*. The Journal of Geology. Volume 40, pp 443-451.