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Solving imperfect market equilibrium problems with convex optimization

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Abstract

The approach of choice to analyze markets with imperfect competition has traditionally been complementarity modeling. In this paper, we show that the majority of equilibrium models in the literature can in fact be cast as convex optimization problems, not requiring the derivation and implementation of Karush-Kuhn-Tucker conditions. This is achieved by adding appropriate terms accounting for market power exertion to the well-known social welfare maximization objective. Resulting models can be solved orders of magnitudes faster using off-the-shelve optimization software compared to solving complementarity problems. We demonstrate a speed-up of factor 118 for a deterministic large-scale multi-period market model, and factor 640 for a stylized multi-stage stochastic market problem, resulting in solving times of minutes instead of tens of hours for typical problem sizes. The presented method will allow to extend existing large-scale optimization problems to address market power exertion with minimal impact on computational tractability. Moreover, the drastically reduced solution times will allow to increase geographical scope and represent economic, technical and other problem characteristics in much more detail in present days' state of the art equilibrium problems with imperfect competition. Another implication is that many problems in the class of multi-level equilibrium problems can be cast as multi-level optimization problems.

1 Introduction.

Equilibria in perfectly-competitive markets and monopoly markets can be found – under mild convexity conditions – using maximization of a single objective: social welfare and monopoly supplier profit, respectively. However, many resource and energy markets are characterized by an imperfect or oligopolistic market structure, wherein each agent maximizes their own objective while market clearing conditions govern the interactions between agents. For such problems, most researchers do not rely on optimization, but commonly formulate equilibrium models cast as complementarity problems, and use specialized solvers, such as PATH (Dirkse and Ferris, 1993; Ferris and Munson, 2000), to find solutions (Abada and Massol, 2011; Abada, Ehrenmann, and Smeers, 2014; Abrell and Weigt, 2012; Boots, Rijkers, and Hobbs, 2004; Day, Hobbs, and Pang, 2002; Devine, Gabriel, and Moryadee, 2016; Egging and Gabriel, 2006; Egging et al., 2008; Egging et al., 2009; Egging and Huppmann, 2012; Gabriel, Zhuang, and Kiet, 2005; Gabriel, Kiet, and Zhuang, 2005; Gabriel and Smeers, 2006; Gabriel, Zhuang, and Egging, 2009; Haftendorn and Holz, 2010; Haftendorn, Holz,

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and Hirschhausen, 2012; Hobbs, 2001; Hobbs and Rijkers, 2004; Hobbs, Rijkers, and Wals, 2004; Holz, Hirschhausen, and Kemfert, 2008; Holz, Richter, and Egging, 2016; Huppmann and Holz, 2009; Huppmann and Holz, 2012; Huppmann and Egging, 2014; Lise, Hobbs, and Oostvoorn, 2008; Moryadee, Gabriel, and Avetisyan, 2014; Moryadee, Gabriel, and Rehulka, 2014; Neuhoff et al., 2005; Neumann, Viehrig, and Weigt, 2009; Zhuang and Gabriel, 2008; Zwart, 2008).

An alternative approach was taken by Spence (1976), Hashimoto (1985), Nagurney (1999), and Hobbs (2001), who have shown that for the special case of Nash-Cournot games with affine inverse demand, and separable, quadratic production cost functions, optimization of a single objective can be used to find equilibria. Further research on tractable approaches for game-theoretic (market) problems was conducted by Blanchet and Carlier (2016) and Goel and Vazirani (2011). Blanchet and Carlier (2016) developed existence and uniqueness conditions for Nash-Cournot equilibria in transportation problems with multiple agents. Goel and Vazirani (2011) model a trader which buys goods from producers and sells them to consumers. Consumers are charged according to their willingness to pay, which is based on piecewise linear concave utility functions. The trader is effectively a monopolist which captures all the consumer surplus by means of first-degree price discrimination. This market can be modeled as a convex program, and its equilibrium can be determined in polynomial time.

Table 1.1 gives an overview over the best practice model formulations (MF) for the different market types: *MF:PM* a monopolistic market, cast as *p*rofit *m*aximization problem; *MF:SW* a perfectly competitive market, cast as a convex *s*ocial *w*elfare maximization problem; and *MF:MCP* an oligopolistic market, wherein multiple agents exert market power, cast as *m*ixed *c*omplementarity *p*roblem (MCP). In this paper, we propose a model formulation which generalizes all the above approaches and is cast as *c*onvex *p*rogram: *MF:CP*. To this end, we extend the results of Spence (1976), Hashimoto (1985), Nagurney (1999), and Hobbs (2001) and show that under mild convexity conditions equilibria of general imperfect market models can be found by solving a convex optimization problem. MF:CP is derived by adding appropriate terms accounting for market power exertion to the well-known maximization function of MF:SW. We provide an economic interpretation of these terms, and show that the solutions to the MF:MCP and MF:CP are identical.

Table 1.1: Overview over existing approaches and best practice model formulations. MF:PM: model formula-tion as profit maximization problem. MF:SW: model formulation as social welfare maximization problem. MF:MCP: model formulation as mixed complementarity problem. MF:CP: model formulation as convex program.

Market type	Formulation as convex optimization problem	Formulation as set of complementarity conditions
Monopoly	MF:PM (best practice)	–
Perfect competition	MF:SW (best practice)	–
Cournot - Oligopoly	Spence (1976), Hashimoto (1985), Nagurney (1999), and Hobbs (2001)	MF:MCP (best practice)
Generalization (incl. CV)	*MF:CP: our proposal*	MF:MCP (best practice)

Moreover, we illustrate the benefits of MF:CP over MF:MCP in terms of ease of derivation, implementation and solution times for two exemplary market settings: The first setting is labeled stochastic resource market model (STO-RM), uses conjectural variations (CV) to allow for hybrid forms of market power exertion, and includes a range of market characteristics such as infrastructure expansions, seasonality and storage, and uncertainty in aspects such as future demand and investment costs (Baltensperger and Egging, 2017). The second market setting is a deterministic version of the Global Gas Model (DE-GGM), which represents production costs by a not quadratic convex function, the so-called *Golombek* cost function (Egging, Holz, and Gabriel, 2010; Egging, 2013; Egging and Holz, 2016; Holz, Richter, and Egging, 2015; Holz, Richter, and Egging, 2016).

This convex optimization formulation to solve the equilibrium problems in question is a contribution in itself, as it does not require the often tedious derivation and implementation of Karush-Kuhn-Tucker (KKT) conditions. Moreover, a broad range of off-the-shelve optimization software can be used, rather than specialized algorithms to solve complementarity problems. The main benefit of the new formulation is, however, that solution times are drastically reduced, which allows larger and more detailed instances of (market) equilibrium models to be solved within a much shorter time frame.

The remainder of this paper is organized as follows. In Section 2, we introduce a stylized commodity market model and derive the MF:PM, MF:SW, MF:MCP, and MF:CP. In Section 3, we provide several model extensions, including seasonality, infrastructure expansions and stochasticity. Section 4 presents solver run-times for the different model formulations for the STO-RM and deterministic version of the GGM (DE-GGM). Section 5 concludes and discusses some broader implications of the drastic improvements in numerical tractability. The Appendix provides an overview of notation used, the model formulation MF:CP including all extensions introduced in Section 3, and its KKT conditions.

2 Mathematical representation of a stylized commodity market.

Consider a network of interconnected nodes, in which economic agents perform different types of activities. Producers produce amounts of resource, consumers have a price dependent demand for the resource. Trading agents (traders) buy the resource from the producers, transport it to other nodes in the network, and sell it to consumers. Service providers offer transportation services of the commodity between nodes. We assume that each economic agent maximizes its profits.

In the following, we first derive mathematical models for the economic a gents. Second, we derive market clearing conditions, which govern the interplay of the traders with the other economic agents. Third, the interactions of traders with other traders is modeled. The models are parametrized such that different types of markets can be represented (perfectly competitive, monopolistic, oligopolistic, or a mix thereof) by varying the number of traders and adjusting the level of market power each trader can exert. Finally, we present a new model, which generalizes the different types of market forms and which is then cast as a convex optimization problem.

2.1 Mathematical representation of economic agents.

2.1.1 Consumers.

Consumers are represented by an affine inverse demand function[1]:

$$W_n^C\left(\sum_{f\in\mathcal{F}(n)} q_{fn}^C\right) := INT_n^C + SLP_n^C \sum_{f\in\mathcal{F}(n)} q_{fn}^C. \tag{2.1}$$

[1]We list the nomenclature in Appendix A.

INT_n^C is the intersect, $SLP_n^C < 0$ the slope, and $\sum_{f \in \mathcal{F}(n)} q_{fn}^C$ the total demand in node n. The function W_n^C represents the price consumers are willing to pay in node n as a function of the total demand in node n.

2.1.2 Traders.

The optimization problem of trader f reads as follows:

$$\max_{q_{fn}^C, q_{fn}^P, q_{fnm}^A} \quad \sum_{n \in \mathcal{C}(f)} P_n^C(\cdot) q_{fn}^C - \sum_{n \in \mathcal{P}(f)} p_n^P q_{fn}^P - \sum_{(n,m) \in \mathcal{A}(f)} p_a^A q_{fnm}^A \tag{2.2}$$

$$\text{s.t.} \quad q_{fn}^P + \sum_{m \in \mathcal{A}(n)} q_{fmn}^A - q_{fn}^C - \sum_{m \in \mathcal{A}(n)} q_{fnm}^A = 0 \quad \left(\phi_{fn}^N\right) \quad \forall n \in \mathcal{N} \tag{2.3}$$

Expression (2.2) is the trader profit consisting of sales revenues minus cost for resource purchases and transportation. q_{fn}^C is the quantity sold by trader f in node n, p_n^P is the unit price the trader pays to the producer in node n to acquire resource amount q_{fn}^P. p_{nm}^A is the market price for transportation services (payable to the transmission system operator; TSO) for using arc (n, m), and q_{fnm}^A is the quantity transported over the arc from node n to m. Equation (2.3) is the volume balance for trader f at each node n: supply by the producer + imports must equal sales to consumers + exports. ϕ_{fn}^N is the corresponding dual variable and can be interpreted as the marginal cost for trader f to sell the amount q_{fn}^C in node n. $P_n^C(\cdot)$ is the *price function the trader considers* when maximizing its profits, and is defined as:

$$P_n^C(\cdot) := \left(1 - \theta_{fn}^C\right) p_n^C + \theta_{fn}^C W_n^C(\cdot), \tag{2.4}$$

p_n^C is the wholesale price (exogenous to the trader). The willingness to pay $W_n^C(\cdot)$ explicitly depends on the total demand $\sum_{f \in \mathcal{F}(n)} q_{fn}^C$, however, the trader f can only influence the q_{fn}^C, whereas the residual demand $\sum_{\hat{f} \in \mathcal{F}(n) \setminus f} q_{\hat{f}n}^C$ is exogenous to trader f. Price p_n^C and willingness to pay $W_n^C(\cdot)$ are weighted by the CV term $\theta_{fn}^C \in [0, 1]$: if $\theta_{fn}^C = 0$, trader f is a price-taker in node n, and if $\theta_{fn}^C = 1$, trader f exerts market power à la Cournot in node n. When appropriate, intermediate values for θ_{fn}^C can be used.

2.1.3 Service providers.

All the service providers, independently of the service they offer, are assumed to be profit-maximizing price takers. $Z \in \{P, A\}$ is a placeholder for the respective service (production P, transportation A), and z is a placeholder (generalized index) for a specific node n or arc nm where the infrastructure is managed by the service provider. Their maximization problem reads as follows:

$$\max_{s_z^Z \geq 0} \quad p_z^Z s_z^Z - c_z^Z s_z^Z \tag{2.5}$$

$$\text{s.t.} \quad \overline{CAP}_z^Z - s_z^Z \geq 0 \quad \left(\lambda_z^Z\right) \tag{2.6}$$

s_z^Z is the quantity contracted by the service provider of type Z, and p_z^Z is the market price, which is exogenous to the service provider and the trader (but not to the model). c_z^Z are the linear costs arising for the service provider Z_z by contracting s_z^Z. We assume that losses associated with the service are compensated by the provider and are factored into the service cost. $\overline{CAP}_z^Z$ is the infrastructure capacity, and λ_z^Z is the dual variable associated with constraint (2.6), representing a congestion fee.

2.2 Mathematical representation of the market clearing conditions.

The interactions between traders and the service providers (including producers), and traders and consumers is governed by market clearing conditions. We ensure that all markets clear by introducing the following balance equations:

$$s_z^Z - \sum_{f \in \mathcal{F}(z)} q_{fz}^Z = 0 \quad \left(p_z^Z\right) \forall z \in Z, \tag{2.7}$$

$$p_n^C - W_n^C(\cdot) = 0 \quad \left(p_n^C\right) \forall n \in \mathcal{N}. \tag{2.8}$$

In the service markets, the total contracted amount s_z^Z equals the sum of flows q_{fz}^Z of all traders f for the respective service Z, and the service market price p_z^Z is the dual variable. In the consumer markets, we establish the price-quantity relationship by demanding that the price p_n^C equals the willingness to pay $W_n^C(\cdot)$, and explicitly set p_n^C as the dual variable.

2.3 Market representations.

2.3.1 Monopoly.

A monopoly can be represented by a market with only one trader f which has maximum market power ($\theta_{fn}^C = 1$) in all nodes n. We take the trader's profit maximization problem Eqs. (2.2) - (2.3) as a starting point and substitute the price function the trader considers $P_n^C(\cdot)$ and the exogenous prices p_z^Z. (i) Since the market power of trader f is $\theta_{fn}^C = 1$, the price function the trader considers is $P_n^C(\theta_{fn}^C = 1) = INT_n^C + SLP_n^C \sum_{f \in \mathcal{F}(n)} q_{fn}^C$. Since service providers are price takers, the price p_z^Z equals the marginal cost of providing the service. The marginal costs comprise the unit cost c_z^Z plus the congestion fee λ_z^Z, hence, $p_z^Z = c_z^Z + \lambda_z^Z$ (see also Baltensperger et al. (2016)). The congestion fee λ_z^Z is exogenous to the trader, however, for the sake of argument here we assume that the trader knows about the driver of the congestion fee: the capacity constraint (2.6). Therefore, we add the capacity constraint (2.6) to the trader's optimization problem and in turn drop the congestion fee in the objective function, resulting in $p_z^Z = c_z^Z$. This leaves us with contracted quantities per service s_z^Z. s_z^Z is known to the trader, since the trader controls all flows (q_{fn}^C and q_{fz}^Z for all services Z). In fact, the following equivalences hold:

$$s_n^P \equiv \sum_{f \in \mathcal{F}(n)} q_{fn}^P \equiv q_{fn}^P, \tag{2.9}$$

$$s_{nm}^A \equiv \sum_{f \in \mathcal{F}(nm)} q_{fnm}^A \equiv q_{fnm}^A. \tag{2.10}$$

Consequentially, we can fully incorporate the services optimization problems and rewrite the trader's profit maximization problem Eqs. (2.2) - (2.3). This leads to the following formulation of the MF:PM:

$$\max_{q^C_{fn}, q^P_{fn}, q^A_{fnm}} \sum_{n \in \mathcal{C}(n)} \left(INT^C_n + SLP^C_n q^C_{fn} \right) \cdot q^C_{fn} \tag{2.11a}$$

$$- \sum_{n \in \mathcal{P}} c^P_n q^P_{fn} \tag{2.11b}$$

$$- \sum_{(n,m) \in \mathcal{A}} c^A_{nm} q^A_{fnm} \tag{2.11c}$$

$$\text{s.t.} \quad q^P_{fn} + \sum_{m \in \mathcal{A}(n)} q^A_{fmn} - q^C_{fn} - \sum_{m \in \mathcal{A}(n)} q^A_{fnm} = 0 \quad \left(\phi^N_{fn} \right) \quad \forall n \in \mathcal{N} \tag{2.11d}$$

$$\overline{CAP}^P_n - q^P_{fn} \geq 0 \quad \left(\lambda^P_n \right) \quad \forall n \in \mathcal{N} \tag{2.11e}$$

$$\overline{CAP}^A_{nm} - q^A_{fnm} \geq 0 \quad \left(\lambda^A_{nm} \right) \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.11f}$$

2.3.2 Perfectly competitive market.

Perfect competition can be represented by a market with a single social welfare-maximizing trader. To this end, we adjust the objective function of the trader (2.2) to maximize social welfare instead of trader profits, by adding consumer surplus. Generally, consumer surplus can be mathematically represented as follows:

$$\sum_{n \in \mathcal{C}(n)} \left(\frac{1}{2} \left(INT^C_n - p^C_n \right) \cdot \sum_{f \in \mathcal{F}(n)} q^C_{fn} \right) \tag{2.12}$$

$$= \frac{1}{2} \sum_{n \in \mathcal{C}(n)} \left(INT^C_n - \left(INT^C_n + SLP^C_n \sum_{f \in \mathcal{F}(n)} q^C_{fn} \right) \right) \cdot \sum_{f \in \mathcal{F}(n)} q^C_{fn} \tag{2.13}$$

$$= \frac{1}{2} \sum_{n \in \mathcal{C}(n)} \left(-SLP^C_n \right) \left(\sum_{f \in \mathcal{F}(n)} q^C_{fn} \right)^2 \tag{2.14}$$

In fact, all substitutions conducted in the monopoly case transfer to the social welfare maximization case. Hence, we can reuse the monopoly trader's optimization problem (2.11) and add the social welfare term (2.14) to the

objective function after substitution of $\sum_{f \in \mathcal{F}(n)} q^C_{fn}$ by q^C_{fn}. The MF:SW reads as follows:

$$\max_{q^C_{fn}, q^P_{fn}, q^A_{fnm}} \sum_{n \in \mathcal{C}(n)} \left(INT^C_n + SLP^C_n q^C_{fn} \right) \cdot q^C_{fn} \tag{2.15a}$$

$$+ \sum_{n \in \mathcal{C}(n)} \frac{1}{2} \left(-SLP^C_n \right) \left(q^C_{fn} \right)^2 \tag{2.15b}$$

$$- \sum_{n \in \mathcal{P}} c^P_n q^P_{fn} \tag{2.15c}$$

$$- \sum_{(n,m)\in\mathcal{A}} c^A_{nm} q^A_{fnm} \tag{2.15d}$$

$$\text{s.t.} \quad q^P_{fn} + \sum_{m\in\mathcal{A}(n)} q^A_{fmn} - q^C_{fn} - \sum_{m\in\mathcal{A}(n)} q^A_{fnm} = 0 \quad \left(\phi^N_{fn}\right) \quad \forall n \in \mathcal{N} \tag{2.15e}$$

$$\overline{CAP}^P_n - q^P_{fn} \geq 0 \quad \left(\lambda^P_n\right) \quad \forall n \in \mathcal{N} \tag{2.15f}$$

$$\overline{CAP}^A_{nm} - q^A_{fnm} \geq 0 \quad \left(\lambda^A_{nm}\right) \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.15g}$$

2.3.3 Oligopoly: best practice representation.

A pure Cournot oligopoly can be represented by multiple traders $f \in F$ exerting a level of market power θ^C_{fn}=1.

There is a general understanding that, in contrast to the perfectly competitive case, oligopolies cannot be reduced to a single optimization problem. Best-practice is to cast an oligopoly as a Nash game, in which all traders maximize their objectives simultaneously. This is equivalent to finding a solution fulfilling all KKT conditions of all optimization problems and the market clearing conditions simultaneously.

In this section, we demonstrate the best-practice approach, before we introduce an equivalent convex optimization problem in Section 2.4.

For the traders' maximization problems (Eqs. (2.2) and (2.3)), the KKT conditions read as follows:

$$0 \leq -p^C_n - \theta^C_{fn} SLP^C_n q^C_{fn} + \phi^N_{fn} \perp q^C_{fn} \geq 0 \quad \forall f \in F, n \in \mathcal{N} \tag{2.16a}$$

$$0 \leq p^P_n - \phi^N_{fn} \perp q^P_{fn} \geq 0 \quad \forall f \in F, n \in \mathcal{N} \tag{2.16b}$$

$$0 \leq p^A_{nm} - \phi^N_{fm} + \phi^N_{fn} \perp q^A_{fnm} \geq 0 \quad \forall f \in F, n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.16c}$$

$$0 \leq q^P_{fn} + \sum_{m\in\mathcal{A}(n)} q^A_{fmn} - q^C_{fn} - \sum_{m\in\mathcal{A}(n)} q^A_{fnm} \perp \phi^N_{fn} \geq 0 \quad \forall f \in F, n \in \mathcal{N} \tag{2.16d}$$

For the service providers' maximization problems (Eqs. (2.5) and (2.6)), the KKT conditions read as follows:

$$0 \leq -p^Z_z + c^Z_z + \lambda^Z_z \perp s^Z_n \geq 0 \quad \forall n \in \mathcal{N} \tag{2.17a}$$

$$0 \leq \overline{CAP}^Z_z - s^Z_z \perp \lambda^Z_z \geq 0 \quad \forall n \in \mathcal{N} \tag{2.17b}$$

We collect the traders' and service providers' KKT conditions (Eqs. (2.16a) - (2.16d) and (2.17a) - (2.17b)) and stack them with the market clearing conditions (2.7) and (2.8). The resulting set of complementarity constraints form an instance of an MCP and can be solved by suitable solvers, such as PATH. We refer to this problem as MF:MCP hereafter.

2.4 Oligopoly: Equivalent formulation as convex optimization problem.

In this section, we present a convex optimization problem MF:CP for imperfect markets with CV. To model perfect competition (Section 2.3.2), we defined a single trader social welfare optimization problem and altered the trader's objective function. Here, we follow a similar approach: we first gather all traders' optimization problems, and then alter the objective function.

We do not, however, reduce the number of traders. The MF:CP reads as follows:

$$\max_{\substack{q^C_{fn}, q^P_{fn}, q^A_{fnm}, \\ s^P_n, s^A_{nm}}} \quad \sum_{n \in \mathcal{C}(n)} \left(INT^C_n + SLP^C_n \sum_{f \in \mathcal{F}(n)} q^C_{fn} \right) \cdot \sum_{f \in \mathcal{F}(n)} q^C_{fn} \tag{2.18a}$$

$$+ \sum_{n \in \mathcal{C}(n)} \frac{1}{2} \left(-SLP^C_n \right) \left(\sum_{f \in \mathcal{F}(n)} q^C_{fn} \right)^2 \tag{2.18b}$$

$$- \sum_{n \in \mathcal{C}(n)} \sum_{f \in \mathcal{F}(n)} \frac{1}{2} \theta^C_{fn} \left(-SLP^C_n \right) \left(q^C_{fn} \right)^2 \tag{2.18c}$$

$$- \sum_{n \in \mathcal{P}} c^P_n s^P_n \tag{2.18d}$$

$$- \sum_{(n,m) \in \mathcal{A}} c^A_{nm} s^A_{nm} \tag{2.18e}$$

$$\text{s.t.} \quad q^P_{fn} + \sum_{m \in \mathcal{A}(n)} q^A_{fmn} - q^C_{fn} - \sum_{m \in \mathcal{A}(n)} q^A_{fnm} = 0 \quad \left(\phi^N_{fn} \right) \quad \forall f \in F, n \in \mathcal{N} \tag{2.18f}$$

$$\overline{CAP}^P_n - s^P_n \geq 0 \quad \left(\lambda^P_n \right) \quad \forall n \in \mathcal{N} \tag{2.18g}$$

$$\overline{CAP}^A_{nm} - s^A_{nm} \geq 0 \quad \left(\lambda^A_{nm} \right) \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.18h}$$

$$s^P_n - \sum_{f \in \mathcal{F}(n)} q^P_{fn} = 0 \quad \left(p^P_n \right) \quad \forall n \in \mathcal{N} \tag{2.18i}$$

$$s^A_{nm} - \sum_{f \in \mathcal{F}(nm)} q^A_{fnm} = 0 \quad \left(p^A_{nm} \right) \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.18j}$$

Many terms of MF:CP (2.18) are identical to the terms of the MF:SW (2.15), except for the fact that $\sum_{f \in \mathcal{F}(n)} q^C_{fn}$ was not substituted by q^C_{fn}, since here we do not reduce the number of traders: Terms (2.18a), (2.18d), and (2.18e) represent the sum of trader surpluses; Term (2.18b) corresponds to the consumer surplus; Equation (2.18f) ensures that the volume balance holds for each trader and node, and Equations (2.18g) - (2.18h) constrain produced and transported volumes. The two big differences to the social welfare maximization problem are on the one hand Term (2.18c), which accounts for the market power exertion of the traders and is explained in detail in Section 2.4.2, and on the other hand Equations (2.18i) - (2.18j), which ensure the markets clear (cf., Equation (2.7)). Since in a setting with multiple traders $s^Z_n = \sum_{f \in \mathcal{F}(n)} q^Z_{zn} \neq q^Z_{zn}$, these additional equations are necessary to establish a relationship between the total contracted volume in link/node z by service Z, s^Z_z, and the volume contracted trader f, q^Z_{fz} [2].

Any meaningful parametrization of Problem (2.18) will include positive intersections INT^C_n and negative slopes SLP^C_n of the inverse demand functions, non-negative cost terms c^Z_z, and positive capacity limits $\overline{CAP}^Z_z$. Hence, Problem (2.18) has a convex objective function with affine constraints constituting a convex, non-empty feasible region. Therefore, Problem (2.18) is convex.

[2] In case the cost functions are affine or separable with respect to q^Z_{fz}, in fact all the s^Z_z can be substituted by $\sum_{f \in \mathcal{F}(z)} q^Z_{fz}$, and all the p^Z_z by $c^Z_z + \lambda^Z_z$ (Baltensperger et al., 2016), which reduces the problem size and may reduce solution times further.

2.4.1 Proof of equivalence.

We prove equivalence of the MF:CP (Problem (2.18)) and the MF:MCP (comprising Equations (2.16a) - (2.16d), (2.17a) - (2.17b), and market clearing conditions (2.7) - (2.8)) by deriving the KKT conditions of MF:CP, and show that they are equivalent to the set of complementarity conditions forming MF:MCP.

The KKT conditions of MF:CP read as follows:

$$0 \leq -\left(INT_n^C + SLP_n^C \sum_{f \in \mathcal{F}(n)} q_{fn}^C\right) - \theta_{fn}^C SLP_n^C q_{fn}^C + \phi_{fn}^N \perp q_{fn}^C \geq 0 \quad \forall f \in F, n \in \mathcal{N} \tag{2.19a}$$

$$0 \leq p_n^P - \phi_{fn}^N \perp q_{fn}^P \geq 0 \quad \forall f \in F, n \in \mathcal{N} \tag{2.19b}$$

$$0 \leq p_{nm}^A - \phi_{fm}^N + \phi_{fn}^N \perp q_{fnm}^A \geq 0 \quad \forall f \in F, n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.19c}$$

$$0 \leq -p_n^P + c_n^P + \lambda_n^P \perp s_n^P \geq 0 \quad \forall n \in \mathcal{N} \tag{2.19d}$$

$$0 \leq -p_{nm}^A + c_{nm}^A + \lambda_{nm}^A \perp s_{nm}^A \geq 0 \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.19e}$$

$$0 \leq q_{fn}^P + \sum_{m \in \mathcal{A}(n)} q_{fmn}^A - q_{fn}^C - \sum_{m \in \mathcal{A}(n)} q_{fnm}^A \perp \phi_{fn}^N \geq 0 \quad \forall f \in F, n \in \mathcal{N} \tag{2.19f}$$

$$0 \leq \overline{CAP}_n^P - s_n^P \perp \lambda_n^P \geq 0 \quad \forall n \in \mathcal{N} \tag{2.19g}$$

$$0 \leq \overline{CAP}_{nm}^A - s_{nm}^A \perp \lambda_{nm}^A \geq 0 \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.19h}$$

$$0 = s_n^P - \sum_{f \in \mathcal{F}(n)} q_{fn}^P \perp p_n^P \ \text{(f.i.s.)} \quad \forall n \in \mathcal{N} \tag{2.19i}$$

$$0 = s_{nm}^A - \sum_{f \in \mathcal{F}(nm)} q_{fnm}^A \perp p_{nm}^A \ \text{(f.i.s.)} \quad \forall n \in \mathcal{N}, m \in \mathcal{A}(n) \tag{2.19j}$$

It is easily verified that the KKT conditions (2.19) are identical to the set of complementarity conditions of MF:MCP after substitution of the wholesale price p_n^C by its definition $INT_n^C + SLP_n^C \sum_{f \in \mathcal{F}(n)} q_{fn}^C$ (Equation (2.8)). Since the KKT conditions of MF:CP are identical to the complementarity conditions of MF:MCP, both problems have identical solutions. Hence, the convex optimization problem MF:CP is equivalent to mixed complementarity problem MF:MCP.

This is a significant finding not just from a theoretical but also from a practical point of view: obtaining an optimal solution to a *convex* optimization problem is generally preferable over finding a feasible solution to a set of *non-convex* complementarity conditions, since solution times tend to be significantly shorter.

2.4.2 Theoretic rationale for the objective function.

Here, we provide a theoretic rationale for the Term (2.18c), which was in fact introduced by Hobbs (2001) but without its origin being explained or clarified. First, note that the consumer surplus (2.18b) reflects the difference between a monopoly problem and a social welfare optimization. It is easily verified that for a monopolist, all $\theta_{fn}^C = 1$, Term (2.18c) cancels out Term (2.18b), and the MF:CP reduces to a monopoly problem. Similarly, in a perfectly competitive market ($\theta_{fn}^C = 0$), Term (2.18c) equals 0 and the MF:CP represents a social welfare maximization problem.

Next, consider an oligopoly market. According to Cournot theory, each supplier assumes the supplies by the competitors as fixed, and engages in a monopoly game on the residual demand curve (c.f., Eq. (2.22)).

$$\text{Original demand curve:} \qquad P_n^C(\cdot) = INT_n^C + SLP_n^C \sum_{f \in \mathcal{F}(n)} q_{fn}^C \tag{2.20}$$

$$\text{Residual demand curve for supplier } \hat{f}: \qquad P_n^{C*}(\cdot) = \left(INT_n^C + SLP_n^C \sum_{f \in \mathcal{F}(n) \setminus \hat{f}} q_{fn}^C \right) + SLP_n^C q_{\hat{f}n}^C \tag{2.21}$$

$$= INT_{\hat{f}n}^{C*}(\cdot) + SLP_n^C q_{\hat{f}n}^C \tag{2.22}$$

Considering the *residual* demand curve, Term (2.18c) defines how much of the *residual* consumer surplus each specific supplier takes into account. A value of $\theta_{fn}^C = 0$ implies that the specific firm has a perfectly-competitive perspective, and maximizes "residual" consumer surplus. In contrast, a value of $\theta_{fn}^C = 1$ means that the specific firm exerts full market power, and completely ignores "*residual*" consumer surplus. Hence, $\sum_{n \in \mathcal{C}(\hat{f})} \frac{1}{2} \theta_{\hat{f}n}^C \left(-SLP_n^C \right) \left(q_{\hat{f}n}^C \right)^2$ is the consumer surplus which trader $\hat{f}$ perceives they can capture; trader $\hat{f}$ exerts market power according to $\theta_{\hat{f}n}^C$ on that. Summing the "residual" consumer surplus term for all traders f results in Term (2.18c) in the objective function.

In conclusion, if $\theta_{fn}^C = 0$ for all traders f and in all nodes n, Problem (2.18) reduces to a social welfare maximization problem. For a single trader, $|F| = 1$, and $\theta_{fn}^C = 1$ for all nodes n, Problem (2.18) specifies a monopoly. For multiple traders $|F| > 1$, and $\theta_{fn}^C = 1$ for all traders f and all nodes n, Problem (2.18) specifies a pure Cournot oligopoly. Other values for θ_{fn}^C as for instance are often assigned in conjectural variation approaches imply that the "residual" consumer surplus term is only partly considered, thereby allowing the representation of hybrid imperfect market structures.

3 Extensions.

The MF:CP above has expanded the Nash-Cournot agent behavior introduced by Hashimoto (1985) in two dimensions: allowing economic agents to exert varying levels of market power by formulating the model as a CV model, and including capacity constraints on production and transport volumes. In this section, we extend the MF:CP to contain multiple stages $s \in \mathcal{S}$, multiple time periods (e.g., seasons) $t \in \mathcal{T}_s$ in each stage s, and several other features. First, in Section 3.2 the MF:CP is extended with storage of resources between time steps within a stage. Next, in Section 3.3 expansion of infrastructure between stages is added. Then we generalize the costs for infrastructure usage and expansions in Section 3.4. Finally, Section 3.5 shows how uncertainty in demand, in investment costs, and in reserves can be included. To highlight the extended problem formulation, the terms associated with the extensions are displayed in different colors. As discussed in Section 3.6, none of these model extensions impedes the equivalence of formulations of the MF:CP and the MF:MCP.

3.1 Multiple stages and time periods.

Problem (2.18) can be extended to multiple stages $s \in \mathcal{S}$ and multiple time periods $t \in \mathcal{T}_s$ within the stages by adding distinct parameters and variables for each stage and time period. To this end, we introduce the subscripts $(\cdot)_{st}$. Similarly, all constraints have to hold for all stages s and time periods t. Finally, the objective function sums over all stages and time periods.

3.2 Storage.

We introduce a new type of profit-maximizing price-taking service provider: the storage operator (SSO). The SSO rents out storage capacity, addition capacity, and extraction capacity to traders. We assume that storage is used to carry over resources between time periods within a stage, but not to carry over resources to the next stage. Fees are charged when adding and extracting.

The SSO is modeled using a similar representation as Eq. (2.5) - (2.6). The unit costs c^I_{ns}, c^X_{ns} are multiplied by the total added and extracted quantities s^I_{nst} and s^X_{nst}. Eq. (3.1g) and (3.1h) ensure that the addition and extraction capacities are not violated, and Eq. (3.1i) guarantees that the total additions are not larger than the capacity of the storage.

The trader problem is extended with additional decision variables, as each trader f controls addition q^I_{fnst} and extraction q^X_{fnst} quantities. The nodal volume balances of the traders (3.1c) are adjusted for storage additions and extractions. Moreover, each trader balances additions and extractions per storage and stage (Eq. (3.1d)). To govern the interaction between SSO and traders (cf., (2.7)), Eq. (3.1l) and (3.1m) ensure all storage service markets clear. These modifications lead to the following (convex!) problem representation (with changes highlighted):[3]

$$\min_{\substack{q^C_{fnst}, q^P_{fnst}, q^A_{fnmst}, \\ s^P_{nst}, s^A_{nmst}, q^I_{fnst}, \\ q^X_{fnst}, s^I_{nst}, s^X_{nst}}} \quad \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}_s} \Big(\text{Terms (2.18b) - (2.18e)} \Big)_{st} \tag{3.1a}$$

$$+ \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}_s} \sum_{n \in \mathcal{N}} \left(c^I_{ns} s^I_{nst} + c^X_{ns} s^X_{nst} \right) \tag{3.1b}$$

$$\text{s.t.} \quad q^P_{fnst} + q^X_{fnst} + \sum_{m \in \mathcal{A}(n)} q^A_{fmnst} - q^C_{fnst} - q^I_{fnst} - \sum_{m \in \mathcal{A}(n)} q^A_{fnmst} = 0 \quad \left(\phi^N_{fnst}\right) \quad \forall f \in \mathcal{F}, n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.1c}$$

$$\sum_{t \in \mathcal{T}_s} q^I_{fnst} - \sum_{t \in \mathcal{T}_s} q^X_{fnst} = 0 \quad \left(\phi^W_{fns}\right) \quad \forall f \in \mathcal{F}, n \in \mathcal{N}, s \in \mathcal{S} \tag{3.1d}$$

$$\overline{CAP}^P_{ns} - s^P_{nst} \geq 0 \quad \left(\lambda^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.1e}$$

$$\overline{CAP}^A_{nms} - s^A_{nmst} \geq 0 \quad \left(\lambda^A_{nmst}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.1f}$$

$$\overline{CAP}^I_{ns} - s^I_{nst} \geq 0 \quad \left(\lambda^I_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.1g}$$

$$\overline{CAP}^X_{ns} - s^X_{nst} \geq 0 \quad \left(\lambda^X_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.1h}$$

$$\overline{CAP}^W_{ns} - \sum_{t \in \mathcal{T}_s} s^I_{nst} \geq 0 \quad \left(\lambda^W_{ns}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S} \tag{3.1i}$$

[3] Here, we assume that all addition periods occur before the extraction periods, and ignore any losses. These features can be easily incorporated.

$$s^P_{nst} - \sum_{f\in\mathcal{F}(n)} q^P_{fnst} = 0 \qquad \left(p^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.1j}$$

$$s^A_{nmst} - \sum_{f\in\mathcal{F}(nm)} q^A_{fnmst} = 0 \qquad \left(p^A_{nmst}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.1k}$$

$$s^I_{nst} - \sum_{f\in\mathcal{F}(n)} q^I_{fnst} = 0 \qquad \left(p^I_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.1l}$$

$$s^X_{nst} - \sum_{f\in\mathcal{F}(n)} q^X_{fnst} = 0 \qquad \left(p^X_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s. \tag{3.1m}$$

3.3 Expansion of infrastructure between stages.

Service operators Z (producers, the SSO, and the TSO) operate their respective infrastructure in entities z (nodes n or links nm). The service operators are modeled as profit-maximizing price-takers, and decide at each stage how much to invest into the expansion of their assets, which results in higher available capacity per the next stage. The added capacity in stage s is denoted $i^{\Delta Z}_{zs}$, the cost of expansion per unit is $c^{\Delta Z}_{zs}$, and the maximum capacity expansion is $\overline{CAP}^{\Delta Z}_{zs}$. After expanding problem (2.18) by these additional terms, the problem reads:[4]

$$\min_{\substack{q^C_{fnst}, q^P_{fnst}, q^A_{fnmst},\\ s^P_{nst}, s^A_{nmst}, i^{\Delta P}_{ns}, i^{\Delta A}_{nms}}} \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}_s} \Big(\text{Terms (2.18b) - (2.18e)} \Big)_{st} \tag{3.2a}$$

$$+ \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} c^{\Delta P}_{ns} i^{\Delta P}_{ns} \tag{3.2b}$$

$$+ \sum_{s\in\mathcal{S}} \sum_{(n,m)\in\mathcal{A}} c^{\Delta A}_{nms} i^{\Delta A}_{nms} \tag{3.2c}$$

$$\text{s.t.} \quad q^P_{fnst} + \sum_{m\in\mathcal{A}(n)} q^A_{fmnst}$$

$$- q^C_{fnst} - \sum_{m\in\mathcal{A}(n)} q^A_{fnmst} = 0 \qquad \left(\phi^N_{fnst}\right) \quad \forall f \in \mathcal{F}, n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.2d}$$

$$\overline{CAP}^P_{ns} + \sum_{s'\in pred(s)} i^{\Delta P}_{ns'} - s^P_{nst} \geq 0 \qquad \left(\lambda^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.2e}$$

$$\overline{CAP}^A_{nms} + \sum_{s'\in pred(s)} i^{\Delta A}_{nms'} - s^A_{nmst} \geq 0 \qquad \left(\lambda^A_{nmst}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.2f}$$

$$\overline{CAP}^{\Delta P}_{ns} - i^{\Delta P}_{ns} \geq 0 \qquad \left(\lambda^{\Delta P}_{ns}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S} \tag{3.2g}$$

$$\overline{CAP}^{\Delta A}_{nms} - i^{\Delta A}_{nms} \geq 0 \qquad \left(\lambda^{\Delta A}_{nms}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S} \tag{3.2h}$$

$$s^P_{nst} - \sum_{f\in\mathcal{F}(n)} q^P_{fnst} = 0 \qquad \left(p^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.2i}$$

$$s^A_{nmst} - \sum_{f\in\mathcal{F}(nm)} q^A_{fnmst} = 0 \qquad \left(p^A_{nmst}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}_s. \tag{3.2j}$$

[4] To limit the lenght of the exposition we refrain from presenting the storage capacity expansions here.

3.4 Non-linear costs for infrastructure usage and infrastructure expansion.

As long as convexity of optimization problem (2.18) is preserved, more general functional forms could be used to represent agent behavior and the network of commodity markets under consideration. Thus far we have assumed linear costs for infrastructure usage and expansions, including for *production services*. Cost functions only affect the structure of the objective function, therefore any convex infrastructure usage costs and expansion costs can be used, including, for instance, the above-mentioned Golombek production cost function often used in natural gas market models. [5] A generalized (but still convex) cost structure can be represented as follows (with changes highlighted):

$$\min_{q^C_{fnst}, q^P_{fnst}, q^A_{fnmst}} \quad \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}_s} \Big(\text{Terms (2.18b) - (2.18a)} \Big)_{st} \tag{3.3a}$$

$$+ \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}_s} \sum_{n\in\mathcal{N}} c^P_{ns}(s^P_{nst}) \tag{3.3b}$$

$$+ \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}_s} \sum_{(n,m)\in\mathcal{A}} c^A_{nms}(s^A_{nmst}) \tag{3.3c}$$

$$\text{s.t.} \quad q^P_{fnst} + \sum_{m\in\mathcal{A}(n)} q^A_{fmnst}$$

$$- q^C_{fnst} - \sum_{m\in\mathcal{A}(n)} q^A_{fnmst} = 0 \quad \left(\phi^N_{fnst}\right) \quad \forall f \in \mathcal{F}, n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.3d}$$

$$\overline{CAP}^P_{ns} - s^P_{nst} \geq 0 \quad \left(\lambda^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.3e}$$

$$\overline{CAP}^A_{nms} - s^A_{nmst} \geq 0 \quad \left(\lambda^A_{nmst}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.3f}$$

$$s^P_{nst} - \sum_{f\in\mathcal{F}(n)} q^P_{fnst} = 0 \quad \left(p^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.3g}$$

$$s^A_{nmst} - \sum_{f\in\mathcal{F}(nm)} q^A_{fnmst} = 0 \quad \left(p^A_{nmst}\right) \quad \forall n \in \mathcal{N}, m \in \mathcal{M}, s \in \mathcal{S}, t \in \mathcal{T}_s, \tag{3.3h}$$

3.5 Input parameter uncertainty.

Uncertainty can be modeled by defining a multi-stage stochastic program representing a scenario tree, with different values for uncertain parameters in each node of the tree, and appropriate probabilities for each scenario (c.f.,(Birge and Louveaux, 2011; Kall and Wallace, 1994)). To this end, we alter the meaning of the notation introduced in Section 3.1). In the following, index $s \in \mathcal{S}$ stands for "scenario tree node" instead of "stage". Scenario nodes are weighed by probabilities γ_s.

To represent reserves uncertainty, condition (3.4g) is introduced. To represent stochastic demand and infrastructure investments no new conditions have to be introduced. This leads to the following problem formulation (with changes highlighted):

$$\min_{q^C_{fnst}, q^P_{fnst}, q^A_{fnmst}} \quad \sum_{s\in\mathcal{S}} \gamma_s \sum_{t\in\mathcal{T}_s} \Big(\text{Terms (2.18b) - (2.18e)} \Big)_{st} \tag{3.4a}$$

[5] Note that if the costs are linear or quadratic with respect to the quantity, the problem at hand remains a quadratic program rather than a (more) general convex program. (Convex) quadratic programs generally enjoy faster solving times because the Hessian remains unchanged and need not be updated throughout the iterations of the solution process, which saves much time.

$$\text{s.t.} \quad q^P_{fnst} + \sum_{m \in \mathcal{A}(n)} q^A_{fmnst}$$

$$- q^C_{fnst} - \sum_{m \in \mathcal{A}(n)} q^A_{fnmst} = 0 \qquad \left(\phi^N_{fnst}\right) \quad \forall f \in \mathcal{F}, n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.4b}$$

$$\overline{CAP}^P_{ns} - s^P_{nst} \geq 0 \qquad \left(\lambda^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.4c}$$

$$\overline{CAP}^A_{nms} - s^A_{nmst} \geq 0 \qquad \left(\lambda^A_{nmst}\right) \quad \forall n \in \mathcal{N}, m \in \mathcal{M}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.4d}$$

$$s^P_{nst} - \sum_{f \in \mathcal{F}(n)} q^P_{fnst} = 0 \qquad \left(p^P_{nst}\right) \quad \forall n \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.4e}$$

$$s^A_{nmst} - \sum_{f \in \mathcal{F}(nm)} q^A_{fnmst} = 0 \qquad \left(p^A_{nmst}\right) \quad \forall (n,m) \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}_s \tag{3.4f}$$

$$\overline{CAP}^{PR}_{ns} - \sum_{s^prime \in \surd \nabla \rceil \lceil (\int)} \sum_{t \in \mathcal{T}_s} s^P_{ns't} \geq 0 \qquad \left(\lambda^{PR}_{ns}\right) \quad \forall n \in \mathcal{N}, s \in \sqcup \rceil \nabla \Updownarrow (\mathcal{S}) \tag{3.4g}$$

3.6 Equivalence of MF:CP and MF:MCP for the extended model.

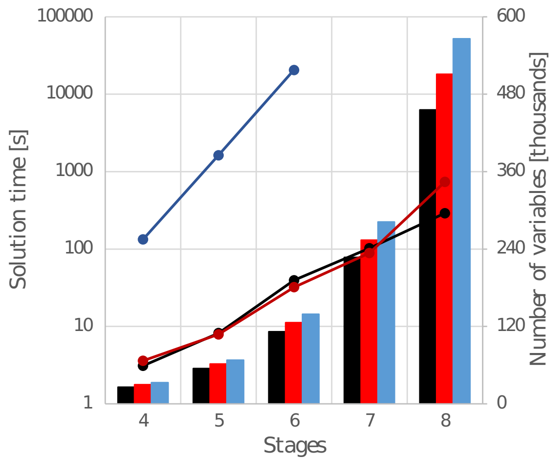
The MF:CP, including all expansions introduced in Section 3, is provided in Appendix B.1. In Appendix B.2, the KKT conditions of the extended MF:CP are presented. The KKT conditions correspond to the extended MF:MCP, hence, none of the extensions void the equivalence of the MF:CP and the MF:MCP. For a derivation of the MF:MCP from the individual agents' optimization problems including extensions we refer to Baltensperger et al. 2018 (2017)[6].

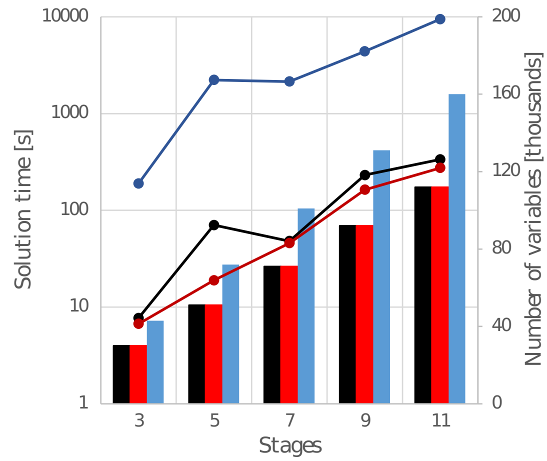
4 Computational impact.

We demonstrate the practical implications of our findings u sing t wo n umerical e xamples. We solve the MF:MCP, MF:CP, and MF:SW of two commodity market models on a personal computer (3.40GHz CPU, 16GB RAM). The STO-RM is a stylized multi-stage stochastic commodity market model consisting of nine interlinked nodes Baltensperger and Egging (2017). STO-RM covers three seasons per stage, inter-seasonal storage, infrastructure expansions, quadratic production costs, and stochastic demand. The DE-GGM is the deterministic version of the Global Gas Model (GGM) (ref. Egging (2010), Egging (2013), Holz, Richter, and Egging (2015), and Holz, Richter, and Egging (2016)), which consists of 90+ nodes and several hundred transmission links. In contrast to STO-RM, the DE-GGM features the convex, non-quadratic Golombek production cost function, and is based on real-world data. It does not, however, include stochasticity. Even so, between the two models most of the extensions introduced in Section 3 are included, and they can both be represented by an adequate parametrization of the optimization problem presented in Appendix B.1.

Both models are implemented in GAMS v24.7.1 (Brooke et al. 1998). The number of stages (and scenarios in the stochastic model) was varied to test how solution times scale with model size increases. The MF:MCP formulations were solved with PATH v4.7.04 (Dirkse and Ferris, 1993; Ferris and Munson, 2000) and default solver settings. For STO-RM, the MF:CP forms an instance of a quadratic program (QP) and to solve the instances, IBM ILOG CPLEX v12.6.3.0 IBM, 2016

[6] The commodity market model introduced by Baltensperger and Egging (2017) includes all extensions introduced in Section 3 except for: storage for four or more seasons; non-linear infrastructure expansion costs; stochastic investment costs; and stochastic reserves.





MF:SW: variables MF:CP: variables MF:MCP: variables MF:SW: sol. time MF:CP: sol. time MF:MCP: sol. time

Figure 4.1: Number of variables and solution times for the STO-RM.

Figure 4.2: Number of variables and solution times for the DE-GGM.

with default solver settings was used. For the DE-GGM, the MF:CP was solved with the non-linear solver Artelys Knitro v10.0.1 (Artelys, 2016) with slightly adapted parameter settings (Table 4.1). Simulations that did not finish within 24 hours (86400 seconds) were aborted.

Table 4.1: Artelys Knitro solver settings. Default values were used except for the settings listed here. *The parameter *pivot* was set to 10^{-7} for solving the MF:SW formulation of the DE-GGM with 11 stages; For all other runs, the standard value of 10^{-8} was used.

Parameter	Value	Default value
algorithm	1	0
bar_murule	5	4
bar_switchrule	1	2
pivot	10^{-7*}	10^{-8}

Figure 4.1 displays the sizes and solution times of the STO-RM instances. The optimization problems MF:CP and MF:SW solve all instances in comparable time, and significantly faster than the MF:MCP. For 6 stages, the MF:MCP takes 640 times as long to solve compared to the MF:CP (five hours and 43 minutes vs. 32 seconds), and for 7 and more stages solution times exceed 24 hours. The MF:CP and MF:SW solve within minutes even for the largest problem sizes. We obtain similar results for the DE-GGM (Figure 4.2). The largest time decrease for the MF:CP compared to the MF:MCP is for the five-stage problem and comprises a factor 118 (37 minutes vs. 19 seconds).

As a side note: the reported variable numbers are indicative, and illustrate the relative model sizes. As expected, the MF:MCP instances are larger than the MF:CP and MF:SW. However, based on the model formulations, the number of variables after pre-solving MF:CP and MF:SW should be the same. We observe this for DE-GGM, but not for STO-RM. This is a consequence of the less lean implementation of STO-RM compared to DE-GGM. For DE-GGM, most variables that will be zero in a solution are fixed to zero before the model is pre-solved, thus the reported number of variables depends less on the pre-solver. In contrast, none of the variables were fixed to zero in the STO-RM variants, and the pre-processor seems to miss out on some model simplifying substitutions.

Overall, the results convincingly show that the MF:CP solves orders of magnitude faster than MF:MCP in all simulations. Additionally, the proposed approach should allow to address market power exertion in (continuous) market models with negligible impact on solving times compared to perfectly competitive models (MF:SW), given the limited impact of the term (2.18c) on the shape of the objective function, and having no impact on the feasible region.

5 Conclusions and implications.

This paper demonstrates the equivalence of a convex problem formulation and an MCP for a general commodity market model with imperfectly competing agents under mild convexity conditions. Thus, the majority of equilibrium models for energy and resource markets in the literature can be cast as convex optimization problems. We show for several instances of a deterministic and stochastic model formulation that the convex problems can be solved orders of magnitudes faster compared to solving the equivalent MCP using off-the-shelve optimization software.

Our findings imply that oligopolistic and CV-based commodity market models can be scaled up in size significantly while maintaining computational tractability. This allows to increase geographical scope and detail, to represent economic, technical and other problem characteristics in much more detail, or include many more scenarios in stochastic problems than is currently the case in state-of-the-art equilibrium model formulations. For instance, we expect that models such as Multimod (Huppmann and Egging, 2014) could represent all main energy consuming and producing countries, as well as more detail in the number of energy technologies and carriers if they would be updated to use the MF:CP.

Furthermore, existing large-scale (continuous) optimization problems can easily be extended to address market power exertion with minimal impact on computational tractability. This is of particular relevance for energy market analysis, where market power plays a crucial role in today's markets, but is often ignored in the modeling and analysis.

Moreover, decomposition techniques can be applied without special modifications towards MCP formulations. For instance, the *standard* Benders Decomposition approach can be used to solve large-scale stochastic market equilibrium problems, instead of the advanced variational inequality-based variants developed and applied by Fuller and Chung (2008), Gabriel and Fuller (2010), and Egging (2013).

Another implication is that many problems in the class of multi-level equilibrium problems (mathematical program with equilibrium constraints (MPEC), equilibrium problem with equilibrium constraints (EPEC)) do in fact belong to the class of multi-level optimization problems. This will not resolve the issue of non-uniqueness, but will help to more rapidly explore the set of solutions.

A Notation.

Table A.1: Sets and indices

Index set	Description
$f \in \mathcal{F}$	Traders
$f \in \mathcal{F}(n)$	Traders that can serve demand at node n
$f \in \mathcal{F}(nm)$	Traders that can contract service at arc (n, m)
$f \in \mathcal{F}(z)$	Traders that can contract services of infrastructure z
$n, m \in \mathcal{N}$	Nodes
$m \in \mathcal{A}(n)$	Neighboring nodes
$n \in \mathcal{C}(n)$	Nodes where consumers are located
$n \in \mathcal{C}(f)$	Nodes where trader f can sell to the consumers
$n \in \mathcal{P}$	Nodes where production finds place
$n \in \mathcal{P}(f)$	Nodes where trader f can buy from the producer
$(n, m) \in \mathcal{A} \subset \mathcal{N} \times \mathcal{N}$	Arcs
$(n, m) \in \mathcal{A}(f)$	Arcs accessible to trader f
$s \in \mathcal{S}$	Stages
$s' \in pred(s)$	Previous stages; predecessor nodes in stochastic scenario tree
$s' \in term(S)$	Terminal stages; terminal nodes in stochastic scenario tree
$t \in \mathcal{T}_s$	Periods (within stages; e.g., seasons)
$t' \in pred\,(t)$	Previous periods within the same stage
$z \in \mathcal{Z}$	Generalized index for infrastructures of type z

Table A.2: Parameters

Parameter	Description
$\overline{CAP}^{\Delta A}_{nms}$	Upper bound on arc capacity expansion
$\overline{CAP}^{\Delta I}_{ns}$	Upper bound on addition capacity expansion
$\overline{CAP}^{\Delta P}_{ns}$	Upper bound on production capacity expansion
$\overline{CAP}^{\Delta W}_{ns}$	Upper bound on storage capacity expansion
$\overline{CAP}^{\Delta X}_{ns}$	Upper bound on extraction capacity expansion
$\overline{CAP}^{\Delta Z}_{ns}$	Upper bound on expansion of capacity of type ‡
$\overline{CAP}^{A}_{nms}$	Upper bound on arc capacity
$\overline{CAP}^{I}_{ns}$	Upper bound on addition capacity
$\overline{CAP}^{P}_{ns}$	Upper bound on production capacity
$\overline{CAP}^{W}_{ns}$	Upper bound on storage capacity
$\overline{CAP}^{X}_{ns}$	Upper bound on extraction capacity
$\overline{CAP}^{Z}_{z}$	Upper bound on capacity of infrastructure type z
$c^{A}_{nms}()$	(Convex) cost of transportation (services) - TSO
$c^{\Delta A}_{nms}()$	(Convex) cost of arc capacity expansion
$c^{\Delta I}_{ns}()$	(Convex) cost of addition capacity expansion
$c^{\Delta P}_{ns}()$	(Convex) cost of production capacity expansion
$c^{\Delta X}_{ns}()$	(Convex) cost of extraction capacity expansion
$c^{\Delta W}_{ns}()$	(Convex) cost of storage capacity expansion
$c^{I}_{ns}()$	(Convex) cost of storage addition (services) - SSO
$c^{P}_{ns}()$	(Convex) cost of production (services) - producer
$c^{X}_{ns}()$	(Convex) cost of storage extraction (services) - SSO
INT^{C}_{nst}	Intercept of the inverse demand curve - strictly positive
SLP^{C}_{nst}	Slope of the inverse demand curve - strictly negative
θ^{C}_{fnst}	conjectural variation parameter - value in $[0, 1]$

Table A.3: Variables

Variable	Description
$i^{\Delta A}_{nms}$	Expansion of arc capacity
$i^{\Delta I}_{ns}$	Expansion of addition capacity
$i^{\Delta P}_{ns}$	Expansion of production capacity
$i^{\Delta W}_{ns}$	Expansion of storage capacity
$i^{\Delta X}_{ns}$	Expansion of extraction capacity
q^{A}_{fnmst}	Transportation volume - trader
q^{C}_{fnst}	Volume sold to consumers - trader
q^{I}_{fnst}	Storage addition volume - trader
q^{P}_{fnst}	Production volume
q^{X}_{fnst}	Storage extraction volume - trader
q^{Z}_{fzst}	Type z infrastructure service volume - trader
s^{A}_{nmst}	(Aggregate) transportation volume TSO
s^{I}_{nst}	(Aggregate) storage addition volume - SSO
s^{P}_{nst}	Production volume producer
s^{X}_{nst}	(Aggregate) storage extraction volume - SSO
s^{Z}_{zst}	(Aggregate) contract volume - service provider (c.f. Equation (2.6))
p^{A}_{nmst}	(Dual) market price for transportation (services)
p^{I}_{nst}	(Dual) market price for storage addition (services)
p^{P}_{nst}	(Dual) market price for production (services)
p^{X}_{nst}	(Dual) market price for storage extraction (services)
p^{Z}_{zst}	(Dual) market price for type z infrastucture (services)
λ^{A}_{nmst}	Dual to transportation arc capacity
$\lambda^{\Delta A}_{nms}$	Dual to arc capacity expansion
$\lambda^{\Delta I}_{ns}$	Dual to addition capacity expansion
$\lambda^{\Delta P}_{ns}$	Dual to production capacity expansion
$\lambda^{\Delta W}_{ns}$	Dual to storage capacity expansion
$\lambda^{\Delta X}_{ns}$	Dual to extraction capacity expansion
λ^{I}_{nst}	Dual to storage addition capacity
λ^{P}_{nst}	Dual to production capacity
λ^{PR}_{ns}	Dual to reserves constraint - producer
λ^{X}_{nst}	Dual to storage extraction capacity
ϕ^{N}_{fnst}	Dual to nodal mass balance constraint - trader
ϕ^{W}_{fns}	Dual to storage mass balance - trader

B Model formulation including all expansions.

This section provides a formulation of the resource model (2.18) after including all expansions introduced in Section 3. The expansions are colorized to highlight their origin: multiple stages $s \in \mathcal{S}$; stochastic demand, infrastructure investment costs, and reserves; storage of resources between seasons, expansions of infrastructure between stages, and non-linear (convex) infrastructure and expansions costs.

B.1 Convex mathematical program for the resource market.

$$\min_{\substack{q^C_{fnst}, q^P_{fnst}, q^A_{fnmst},\\ s^P_{nst}, s^A_{nmst}, q^I_{fnst}, q^X_{fnst},\\ s^I_{nst}, s^X_{nst}, i^{\Delta P}_{nst},\\ i^{\Delta A}_{nmst}, i^{\Delta I}_{nst}, i^{\Delta X}_{nst}, i^{\Delta W}_{nst}}} \quad - \sum_{s\in\mathcal{S}} \gamma_s \sum_{t\in\mathcal{T}_s} \sum_{n\in\mathcal{C}(f)} \frac{1}{2}(-SLP^C_{nst}) \left(\sum_{f\in\mathcal{F}(n)} q^C_{fnst} \right)^2 \tag{B.1a}$$

$$+ \sum_{s\in\mathcal{S}} \gamma_s \sum_{t\in\mathcal{T}_s} \sum_{n\in\mathcal{C}(f)} \sum_{f\in\mathcal{F}(n)} \frac{1}{2}\theta^C_{fnst}(-SLP^C_{nst}) \left(q^C_{fnst}\right)^2 \tag{B.1b}$$

$$- \sum_{s\in\mathcal{S}} \gamma_s \sum_{t\in\mathcal{T}_s} \sum_{n\in\mathcal{C}(f)} \sum_{f\in\mathcal{F}(n)} \left(INT^C_{nst} + SLP^C_{nst} \sum_{f\in\mathcal{F}(n)} q^C_{fnst} \right) q^C_{fnst} \tag{B.1c}$$

$$+ \sum_{s\in\mathcal{S}} \gamma_s \sum_{n\in\mathcal{N}} \left(\sum_{t\in\mathcal{T}_s} c^P_{ns}(s^P_{nst}) + c^{\Delta P}_{ns}(i^{\Delta P}_{ns}) \right) \tag{B.1d}$$

$$+ \sum_{s\in\mathcal{S}} \gamma_s \sum_{nm\in\mathcal{A}} \left(\sum_{t\in\mathcal{T}_s} c^A_{nms}(s^A_{nmst}) + c^{\Delta A}_{nms}(i^{\Delta A}_{nms}) \right) \tag{B.1e}$$

$$+ \sum_{s\in\mathcal{S}} \gamma_s \sum_{n\in\mathcal{N}} \left(\sum_{t\in\mathcal{T}_s} c^I_{ns}(s^I_{nst}) + c^{\Delta I}_{ns}(i^{\Delta I}_{ns}) \right) \tag{B.1f}$$

$$+ \sum_{s\in\mathcal{S}} \gamma_s \sum_{n\in\mathcal{N}} \left(\sum_{t\in\mathcal{T}_s} c^X_{ns}(s^X_{nst}) + c^{\Delta X}_{ns}(i^{\Delta X}_{ns}) \right) \tag{B.1g}$$

$$+ \sum_{s\in\mathcal{S}} \gamma_s \sum_{n\in\mathcal{N}} c^{\Delta W}_{ns}(i^{\Delta W}_{ns}) \tag{B.1h}$$

subject to:

volume balances:

$$q^P_{fnst} + q^X_{fnst} + \sum_{m\in\mathcal{A}(n)} q^A_{fmnst}$$

$$- q^C_{fnst} - q^I_{fnst} - \sum_{m\in\mathcal{A}(n)} q^A_{fnmst} = 0 \qquad \left(\phi^N_{fnst}\right) \quad \forall f, n, s, t \tag{B.2a}$$

$$\sum_{t\in\mathcal{T}_s} q^I_{fnst} - \sum_{t\in\mathcal{T}_s} q^X_{fnst} = 0 \qquad \left(\phi^W_{fns}\right) \quad \forall f, n, s \tag{B.2b}$$

Capacity restrictions:

$$\overline{CAP}^{P}_{ns} + \sum_{s' \in pred(s)} i^{\Delta P}_{ns'} - s^{P}_{nst} \geq 0 \qquad \left(\lambda^{P}_{nst}\right) \quad \forall n, s, t \tag{B.2c}$$

$$\overline{CAP}^{PR}_{ns} - \sum_{s' \in pred(s)} \sum_{t \in \mathcal{T}_s} s^{P}_{ns't} \geq 0 \qquad \left(\lambda^{PR}_{ns}\right) \quad \forall n, s \in term\left(\mathcal{S}\right) \tag{B.2d}$$

$$\overline{CAP}^{A}_{nms} + \sum_{s' \in pred(s)} i^{\Delta A}_{nms'} - s^{A}_{nmst} \geq 0 \qquad \left(\lambda^{A}_{nms}\right) \quad \forall n, m, s, t \tag{B.2e}$$

$$\overline{CAP}^{I}_{ns} + \sum_{s' \in pred(s)} i^{\Delta I}_{ns'} - s^{I}_{nst} \geq 0 \qquad \left(\lambda^{I}_{nst}\right) \quad \forall n, s, t \tag{B.2f}$$

$$\overline{CAP}^{X}_{ns} + \sum_{s' \in pred(s)} i^{\Delta X}_{ns'} - s^{X}_{nst} \geq 0 \qquad \left(\lambda^{X}_{nst}\right) \quad \forall n, s, t \tag{B.2g}$$

$$\overline{CAP}^{W}_{ns} + \sum_{s' \in pred(s)} i^{\Delta W}_{ns'} - \sum_{t' \in \mathcal{T}_s} \left(s^{I}_{nst'}\right) \geq 0 \qquad \left(\overline{\lambda}^{W}_{ns}\right) \quad \forall n, s \tag{B.2h}$$

market clearing conditions:

$$s^{P}_{nst} - \sum_{f \in \mathcal{F}(n)} q^{P}_{fnst} = 0 \qquad \left(p^{P}_{nst}\right) \quad \forall n, s, t \tag{B.2i}$$

$$s^{S}_{nmst} - \sum_{f \in \mathcal{F}(nm)} q^{A}_{fnmst} = 0 \qquad \left(p^{A}_{nms}\right) \quad \forall n, m, s, t \tag{B.2j}$$

$$s^{I}_{nst} - \sum_{f \in \mathcal{F}(n)} q^{I}_{fnst} = 0 \qquad \left(p^{I}_{ns,t}\right) \quad \forall n, s, t \tag{B.2k}$$

$$s^{X}_{nst} - \sum_{f \in \mathcal{F}(n)} q^{X}_{fnst} = 0 \qquad \left(p^{X}_{ns,t}\right) \quad \forall n, s, t \tag{B.2l}$$

restrictions on capacity expansions:

$$\overline{CAP}^{\Delta P}_{ns} - i^{\Delta P}_{ns} \geq 0 \qquad \left(\lambda^{\Delta P}_{ns}\right) \quad \forall n, s \tag{B.2m}$$

$$\overline{CAP}^{\Delta A}_{nms} - i^{\Delta A}_{nms} \geq 0 \qquad \left(\lambda^{\Delta A}_{nms}\right) \quad \forall n, m, s \tag{B.2n}$$

$$\overline{CAP}^{\Delta I}_{ns} - i^{\Delta I}_{ns} \geq 0 \qquad \left(\lambda^{\Delta I}_{ns}\right) \quad \forall n, s \tag{B.2o}$$

$$\overline{CAP}^{\Delta X}_{ns} - i^{\Delta X}_{ns} \geq 0 \qquad \left(\lambda^{\Delta X}_{ns}\right) \quad \forall n, s \tag{B.2p}$$

$$\overline{CAP}^{\Delta W}_{ns} - i^{\Delta W}_{ns} \geq 0 \qquad \left(\lambda^{\Delta W}_{ns}\right) \quad \forall n, s \tag{B.2q}$$

B.2 Corresponding KKT conditions.

Stationarity conditions:

$$\begin{aligned} 0 \leq \quad & -\left(INT^{C}_{nst} + \sum_{f \in \mathcal{F}(n)} SLP^{C}_{nst} q^{C}_{fnst}\right) \\ & + \theta^{C}_{fnst}\left(-SLP^{C}_{nst}\right) q^{C}_{fnst} + \phi^{N}_{fnst} \quad \perp q^{C}_{fnst} \quad \geq 0 \quad \forall f, n, s, t \end{aligned} \tag{B.3a}$$

$$0 \leq \quad p^{P}_{nst} - \phi^{N}_{fnst} \perp q^{P}_{fnst} \quad \geq 0 \quad \forall f, n, s, t \tag{B.3b}$$

$$0 \leq \quad p^{A}_{nmst} - \phi^{N}_{fmst} + \phi^{N}_{fnst} \perp q^{A}_{fnmst} \geq 0 \quad \forall f, n, m, s, t \tag{B.3c}$$

$$0 \leq \quad p^{I}_{nst} + \phi^{N}_{fnst} - \phi^{W}_{fns} \perp q^{I}_{fnst} \quad \geq 0 \quad \forall f, n, s, t \tag{B.3d}$$

$$0 \leq \quad p^X_{nst} - \phi^N_{fnst} + \phi^W_{fns} \perp q^X_{fnst} \quad \geq 0 \quad \forall f, n, s, t \tag{B.3e}$$

$$0 = \quad \frac{dc^P_{ns}(s^P_{nst})}{ds^P_{nst}} + \lambda^P_{nst} + \lambda^{PR}_{ns} - p^P_{nst} \perp s^P_{nst} \quad \text{(free)} \quad \forall f, n, s, t \tag{B.3f}$$

$$0 = \quad \frac{dc^A_{nms}(s^A_{nmst})}{ds^A_{nmst}} + \lambda^A_{nmst} - p^A_{nmst} \perp s^A_{nmst} \quad \text{(free)} \quad \forall f, n, m, s, t \tag{B.3g}$$

$$0 = \quad \frac{dc^I_{ns}(s^I_{nst})}{ds^I_{nst}} + \lambda^I_{nst} + p^I_{nst} \perp s^I_{nst} \quad \text{(free)} \quad \forall f, n, s, t \tag{B.3h}$$

$$0 = \quad \frac{dc^X_{ns}(s^X_{nst})}{ds^X_{nst}} + \lambda^X_{nst} - p^X_{nst} \perp s^X_{nst} \quad \text{(free)} \quad \forall f, n, s, t \tag{B.3i}$$

$$0 \leq \quad \frac{dc^{\Delta P}_{ns}(i^{\Delta P}_{ns})}{di^{\Delta P}_{ns}} + \lambda^{\Delta P}_{ns} - \sum_{s' \in succ(s)} \lambda^P_{nst} \perp i^{\Delta P}_{fns} \quad \geq 0 \quad \forall n, s \tag{B.3j}$$

$$0 \leq \quad \frac{dc^{\Delta A}_{nms}(i^{\Delta A}_{nms})}{di^{\Delta A}_{nms}} + \lambda^{\Delta A}_{nms} - \sum_{s' \in succ(s)} \lambda^A_{nms'} \perp i^{\Delta A}_{fnms} \quad \geq 0 \quad \forall n, m, s \tag{B.3k}$$

$$0 \leq \quad \frac{dc^{\Delta I}_{ns}(i^{\Delta I}_{ns})}{di^{\Delta I}_{ns}} + \lambda^{\Delta I}_{ns} - \sum_{s' \in succ(s)} \lambda^I_{nst} \perp i^{\Delta I}_{fns} \quad \geq 0 \quad \forall n, s \tag{B.3l}$$

$$0 \leq \quad \frac{dc^{\Delta X}_{ns}(i^{\Delta X}_{ns})}{di^{\Delta X}_{ns}} + \lambda^{\Delta X}_{ns} - \sum_{s' \in succ(s)} \lambda^X_{nst} \perp i^{\Delta X}_{fns} \quad \geq 0 \quad \forall n, s \tag{B.3m}$$

$$0 \leq \quad \frac{dc^{\Delta W}_{ns}(i^{\Delta W}_{ns})}{di^{\Delta W}_{ns}} + \lambda^{\Delta W}_{ns} - \sum_{s' \in succ(s)} \lambda^W_{nst} \perp i^{\Delta W}_{fns} \quad \geq 0 \quad \forall n, s \tag{B.3n}$$

Volume balances:

$$0 = q^P_{fnst} + q^X_{fnst} \sum_{m \in \mathcal{A}(n)} q^A_{fmnst} - q^C_{fnst} - q^I_{fnst} - \sum_{m \in \mathcal{A}(n)} q^A_{fnmst} \perp \phi^N_{fnst} \text{(free)} \quad \forall f, n, s, t \tag{B.4a}$$

$$0 = \quad \sum_{t \in \mathcal{T}_s} q^I_{fnst} - \sum_{t \in \mathcal{T}_s} q^X_{fnst} \perp \phi^W_{fns} \quad \text{(free)} \quad \forall f, n, s \tag{B.4b}$$

capacity restrictions on flows:

$$0 \leq \quad \overline{CAP}^P_{ns} + \sum_{s' \in pred(s)} i^{\Delta P}_{ns'} - s^P_{nst} \perp \lambda^P_{nst} \quad \geq 0 \quad \forall n, s, t \tag{B.4c}$$

$$0 \leq \quad \overline{CAP}^{PR}_{ns} - \sum_{s' \in pred(s)} s^P_{ns't} \perp \lambda^{PR}_{ns} \quad \geq 0 \quad \forall n, s \in term(\mathcal{S}) \tag{B.4d}$$

$$0 \leq \quad \overline{CAP}^A_{nms} + \sum_{s' \in pred(s)} i^{\Delta A}_{nms'} - s^A_{nmst} \perp \lambda^A_{nms} \geq 0 \quad \forall n, m, s, t \tag{B.4e}$$

$$0 \leq \quad \overline{CAP}^I_{ns} + \sum_{s' \in pred(s)} i^{\Delta I}_{ns'} - s^I_{nst} \perp \lambda^I_{nst} \quad \geq 0 \quad \forall n, s, t \tag{B.4f}$$

$$0 \leq \quad \overline{CAP}^X_{ns} + \sum_{s' \in pred(s)} i^{\Delta X}_{ns'} - s^X_{nst} \perp \lambda^X_{nst} \quad \geq 0 \quad \forall n, s, t \tag{B.4g}$$

$$0 \leq \quad \overline{CAP}^W_{ns} + \sum_{s' \in pred(s)} i^{\Delta W}_{ns'} - \sum_{t' \in \mathcal{T}_s} \left(s^I_{nst'}\right) \perp \overline{\lambda}^W_{ns} \quad \geq 0 \quad \forall n, s, t \tag{B.4h}$$

(B.4i)

Market clearing conditions:

$$0 = s^P_{nst} - \sum_{f \in \mathcal{F}(n)} q^P_{fnst} \perp p^P_{nst} \text{ (free)} \quad \forall n, s, t \tag{B.5a}$$

$$0 = s^S_{nmst} - \sum_{f \in \mathcal{F}(nm)} q^A_{fnmst} \perp p^A_{nms} \text{ (free)} \quad \forall n, m, s, t \tag{B.5b}$$

$$0 = s^I_{nst} - \sum_{f \in \mathcal{F}(n)} q^I_{fnst} \perp p^I_{ns,t} \text{ (free)} \quad \forall n, s, t \tag{B.5c}$$

$$0 = s^X_{nst} - \sum_{f \in \mathcal{F}(n)} q^X_{fnst} \perp p^X_{ns,t} \text{ (free)} \quad \forall n, s, t \tag{B.5d}$$

Capacity restrictions on investments:

$$0 \leq \overline{CAP}^{\Delta P}_{ns} - i^{\Delta P}_{ns} \perp \lambda^{\Delta P}_{ns} \geq 0 \quad \forall n, s \tag{B.6a}$$

$$0 \leq \overline{CAP}^{\Delta A}_{nms} - i^{\Delta A}_{nms} \perp \lambda^{\Delta A}_{nms} \geq 0 \quad \forall n, m, s \tag{B.6b}$$

$$0 \leq \overline{CAP}^{\Delta I}_{ns} - i^{\Delta I}_{ns} \perp \lambda^{\Delta I}_{ns} \geq 0 \quad \forall n, s \tag{B.6c}$$

$$0 \leq \overline{CAP}^{\Delta X}_{ns} - i^{\Delta X}_{ns} \perp \lambda^{\Delta X}_{ns} \geq 0 \quad \forall n, s \tag{B.6d}$$

$$0 \leq \overline{CAP}^{\Delta W}_{ns} - i^{\Delta W}_{ns} \perp \lambda^{\Delta W}_{ns} \geq 0 \quad \forall n, s \tag{B.6e}$$

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