Engineering Students Learning Mathematics in Context

Frode Rønning

Department of Mathematical Sciences, Norwegian University of Science and Technology, Norway

Abstract: This paper is based on a project where mathematics is taught to engineering students with the aim of obtaining a closer connection between mathematics and engineering subjects – teaching mathematics in context. It is assumed that this will increase the perceived relevance of mathematics for the students. The paper will discuss opportunities and challenges involved when implementing a contextual approach to mathematics. Examples from electrical engineering are shown, and some results from surveys addressing students' perceived relevance of mathematics are presented.

Key words:
Mathematics for engineers, contextual learning, relevance of mathematics

1 Mathematics in Engineering Education

1.1 Connections Between Mathematics and User Programmes

Mathematics has for a long time been recognised as a central subject in engineering education, but there are different views as to what kind of mathematics engineering students should learn, and who should be teaching mathematics to them (Alpers, 2020; Bajpaj, 1985; Scanlan, 1985). In the last 20-30 years, there has been a rapid development in research on mathematics as a service subject, in particular in engineering education (Alpers, 2020). In an ICMI study from 1988, Howson et al. (as cited in Alpers, 2020, pp. 5–6) discuss three main questions regarding mathematics as a service subject, "Why?", "What?" and "How?" Possible answers to "Why?" include enabling students to use techniques and concepts in application subjects, to make use of technology, to "read" mathematics for continuing professional development (learn mathematics as a language), and to have a mathematical "mode of thought". To achieve this, cooperation between mathematicians and colleagues from application subjects was recommended (Alpers, 2020, p. 6).

Although more than 25 years have passed since the ICMI study, it can still be said that the connection between mathematics and user programmes is not well developed. Several recent studies show that there is a lack of connection between mathematics and engineering, and that students perceive the mathematics they learn as not relevant (Faulkner et al., 2019, 2020; Flegg et al., 2012; González-Martín & Hernandes-Gomes, 2017; Harris et al., 2015; Loch & Lamborn, 2016). Similar issues have been identified also with other user programmes, e.g., economics (see Landgärds-Tarvoll, 2024; Landgärds-Tarvoll & Göller, 2024).

1.2 Recent Trends in Engineering Education

Over the last 20 years, the so-called CDIO[1] initiative has had a strong impact on engineering education. A central idea in CDIO is that of an integrated curriculum, which also can be connected to the principle of contextual learning (Crawley et al., 2014, p. 33). CDIO also explicitely addresses mathematics in engineering education through the standard for Simulation-based mathematics: "Engineering programs for which the mathematics curriculum is infused with programming, numerical modelling and simulation from the start" (CDIO, 2022).

An NTNU, there is an ongoing process of revising the technology programmes, called Technology Studies for the Future (FTS[2], 2022), strongly inspired by CDIO ideas. In addition to creating stronger links between mathematics and the various engineering programmes, the revision also takes into account that the way mathematics is used in engineering has changed. Some topics have become more important, while others may be less important. Overall, the emphasis on computational methods and numerical analysis will be strengthened, in alignment with CDIO.

---

[1] CDIO: Conceive, Design, Implement, Operate (http://www.cdio.org)
[2] Fremtidens teknologistudier

At universities with large engineering programmes, mathematics is often taught in a rather generic way, although there are examples of mathematics being designed and taught particularly for specific engineering programmes (see e.g., Alpers, 2008, Enelund et al., 2011). At NTNU, a pilot project for FTS was designed (MARTA[3]) where mathematics was taught with the intention of maintaining a close relation to the engineering programmes involved (Bolstad et al., 2022; Rønning, 2022, 2023b). The project started with only one engineering programme, Electronic Systems Design and Innovation (MTELSYS) and was later extended to also include Cybernetics and Robotics (MTTK) and Industrial Chemistry and Biotechnology (MTKJ). The aim of the project was formulated as to strengthen the engineering students' learning output by developing student active ways of learning and accompanying assessment methods, based on seeing mathematics and engineering in connection, as well as to strengthen the students' perceived relevance of mathematics for their chosen study programme (Rønning, 2023a, p. 2).

For MARTA, four principles were formulated: Developing conceptual knowledge, Developing stronger integration between mathematics and engineering subjects, Developing student active ways of working and new assessment forms, and Developing a programme driven education (Rønning, 2023a, p. 4). These principles can be linked to the FTS principles 1, 3, 4 and 6 (see FTS, 2022, p. 11). For this paper, the principle of stronger integration between mathematics and engineering subjects will be the most central. This can also be linked to the CDIO principle of contextual learning (Crawley et al., 2014, p. 33). The questions that will be addressed in the paper are

1. What challenges and opportunites can be identified when implementing contextual learning in mathematics and engineering?
2. How do the students perceive the relevance of mathematics for their study programme?

For the first question, I will show examples indicating opportunities for integration and discuss their limitations, partly based on data from interviews and more informal discussions with both the mathematics teacher and an engineering teacher. For the second question, I base my discussion on results from student surveys where different questions pertaining to perceived relevance were asked.

2 Contextualised Mathematics

2.1 Modelling Electric Circuits

In electrical engineering, electric circuits is a basic topic which involves the use of mathematics. Textbooks for electrical engineering contain numerous examples and excercises involving modelling of circuits, e.g., RLC-circuits. Such circuits can be modelled with a linear second order differential equation with constant coefficients (see e.g., Nilsson & Riedel, 2011). In these cases, the mathematics is simple in the sense that it is possible to find an analytic solution. However, there are relevant examples of circuits with a non-linear behaviour, e.g., a circuit containing a light emitting diode (LED), a

[3] Matematikk som redskap for tanken (Mathematics as a thinking tool)

voltage source and a resistor. The LED introduces a non-linear expression for the current, which makes the mathematics more complicated (see Bolstad et al., 2022).

Figure 1 shows an RLC-circuit which also contains an amplifier, characterised by a constant G. This circuit can be modelled with the linear differential equation (1) (Lundheim, 2021), where y is the voltage.

$$(1)\ y'' + (1-G)\frac{R}{L}y' + \frac{1}{LC}y = 0$$

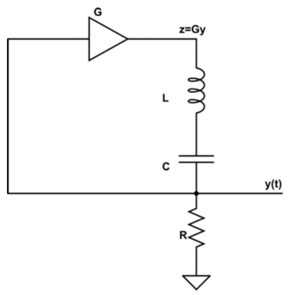

Figure 1. A circuit with an amplifier

If $G = 1$, the equation (1) has the form $y'' = -k^2y$, with solutions $y = \sin(kx)$ and $y = \cos(kx)$, representing stable oscillations. This is what the engineer wants, but keeping $G = 1$ is impossible in practice. Therefore the circuit is modified by introducing non-linear components, resulting in a non-linear differential equation which does not have an analytic solution (see Rønning, 2023b). In the electronics course, the circuit can be built, and the output can be measured and visualised on an oscilloscope, and further compared to the results from the mathematical computations.

An important point to be made from these examples is that realistic, but still simple, engineering applications may lead to the need for mathematical methods that otherwise might not have been introduced, or perhaps would have been considered irrelevant by the students.

2.2 Issues with contextualised mathematics

To include the examples presented in 2.1 in mathematics requires knowledge from electrical engineering. It is desirable that the students actually can build the circuits and do measurements on them that can be compared with the results from mathematical computations. In the first phase of MARTA the connection to engineering was possible since the only engineering programme involved was MTELSYS. In the second phase, including also MTTK and MTKJ, this became more challenging. In an interview, the mathematics teacher in MARTA said that finding relevant examples from chemistry in the first year was challenging, firstly because he lacked knowledge in chemistry, and secondly, because good examples would require more background both in mathematics and in chemisty than the students had at the time. Electric circuits do not require so much background knowledge, and therefore the teacher used electric circuits as examples also for the mixed group of students, realising that by this approach a part of the student group might not find the examples that relevant. He also said that for the mixed group of students, the presentation became more general, or "more mathematical". However, a model with specialised mathematics courses for each engineering programme at universities like NTNU with a very large and diverse engineering education would not be economically feasible. Therefore, when scaling up the MARTA project, certain concessions have to be made.

3 Revising the Mathematics Courses to align with FTS

3.1 Developing the revised course portfolio

The development of the revised course portfolio has taken place in collaboration with the user programmes to decide what mathematical topics are most relevant for the different programmes and at what point in time it is desirable to introduce a certain topic. This has resulted in a splitting of the programmes in four clusters where the sequence and emphasis of topics vary between the clusters. A common feature for all clusters is increased emphasis on computational mathematics and also on linear algebra. Other topics, like complex analysis, may be reduced or taken out for some of the programmes where this topic traditionally has been included.

3.2 Differences between programmes in perceived relevance

In 2022, 2023 and 2024, identical questionnaires were administered to all first-year student in the Master of Technology (siv.ing.) programmes where some items addressed the connection between mathematics and engineering. The results indicate that there seems to be a difference between study programmes regarding the perceived relevance of mathematics, and also regarding the extent to which mathematics is visible in the engineering subjects. Table 1 shows some results regarding Item A: “I don’t think the mathematics I have learned is very relevant for my study programme”. Here OTHER means all study programmes not part of MARTA. All the numbers are from 2023 where MARTA included MTELSYS, MTTK and MTKJ. Here, MTTK is significantly (shown by Fisher’s exact test) different from MTKJ and OTHER, and also MTKJ shows significantly lower relevance than OTHER.

	Agree (%)	Disagree (%)
MTKJ (n = 37)	46.0 (17)	54.0 (20)
MTTK (n = 39)	10.3 (4)	89.7 (35)
OTHER (n = 472)	26.7 (126)	72.3 (346)

Table 1: Item A: “I don’t think the mathematics I have learned is very relevant for my study programme”.

The perceived relevance could be connected to the extent to which teachers in engineering subjects point to the importance of mathematics. This is addressed in Item B: “Teachers in other subjects (not mathematics subjects) have made the importance of learning mathematics visible”. Here, MTTK scores significantly better than both MTKJ and OTHER, also shown by Fisher’s exact test.

	Agree (%)	Disagree (%)
MTKJ (*n* = 37)	70.3 (26)	29.7 (11)
MTTK (*n* = 40)	95.0 (38)	5.0 (2)
OTHER (*n* = 472)	64.0 (302)	26.0 (170)

Table 2: Item B: “Teachers in other subjects (not mathematics subjects) have made the importance of learning mathematics visible”.

It is important to emphasise that these results reflect the students’ perception after their first year of study. It may very well be that for some programmes, the importance of mathematics will be visible later.

4 Discussion

In this brief presentation I have shown that teaching mathematics in context is indeed possible, and examples from engineering may trigger the introduction of mathematical topics and methods that otherwise might not have been included in the mathematics courses. However, if the students cannot relate to the examples, the examples will not have the desired effect. Therefore, it is recommended that the students are split according to their engineering programme, but to what extent this may be accomplished is a matter of available resources. To be able to introduce realistic examples requires a close collaboration between teachers in mathematics and engineering subjects. There is evidence to suggest that there are differences between engineering programmes regarding to what extent the importance of mathematics is made visible at an early stage, and there may be good reasons for such differences. Such differences have also been observed by other researches (Faulkner et al., 2020; González-Martín et al., 2021). Lack of connections could lead to loss of motivation and further to drop-out (Faulkner et al., 2019; Flegg et al., 2012). Drop-out often happens early, and therefore it can be argued that it is important to make the connection between mathematics and engineering early. There is also evidence to show that if the mathematics that is learnt early is only used at a later stage, the students struggle to make use of it (Faulkner et al., 2019; Harris et al., 2015).

References

Alpers, B. (2008). The mathematical expertise of mechanical engineers: The case of machine element dimensioning. In B. Alpers, S. Hibberd, D. Lawson, L. Mustoe, & C. Robinson (Eds.), *Proceedings of the 14th SEFI-MWG European Seminar on Mathematics in Engineering Education, Loughborough, 6-9 April 2008* (pp. 1–7). SEFI.

Alpers, B. (2020). *Mathematics as a service subject at the tertiary level. A state-of-the-art report for the Mathematics Interest Group*. European Society for Engineering Education (SEFI).

Bajpai, A. C. (1985). The role of mathematics in engineering education: A mathematician’s view. *International Journal of Mathematical Education in Science and Technology, 16*(3), 417–430.

Bolstad, T., Høyvik, I.-M., Lundheim, L., Nome, M., & Rønning, F. (2022). Study programme driven engineering education: Interplay between mathematics and engineering subjects. *Teaching Mathematics and its Applications: An International Journal of the IMA, 41*(4), 329–344. https://doi.org/10.1093/teamat/hrac010

CDIO. (2022). *CDIO. Optional standards 3.0*. http://www.cdio.org/content/cdio-optional-standards-30
Crawley, E. F., Malmqvist, J., Östlund, S., Brodeur, D. R., & Edström, K. (2014). *Rethinking engineering education. The CDIO approach* (2nd ed.). Springer.
Enelund, M., Larsson, S., & Malmqvist, J. (2011). Integration of a computational mathematics education in the mechanical engineering curriculum. In P. M. Hussmann (Ed.), *Conference proceedings 7th International CDIO Conference, Technical University of Denmark, 20th - 23th June 2011* (pp. 996–1012). Technical University of Denmark.
Faulkner, B., Earl, K., & Herman, G. (2019). Mathematical maturity for engineering students. *International Journal for Research in Undergraduate Mathematics Education, 5*(1), 97–128. https://doi.org/10.1007/s40753-019-00083-8
Faulkner, B., Johnson-Glauch, N., San Choi, D., & Herman, G. L. (2020). When am I ever going to use this? An investigation of the calculus content of core engineering courses. *Journal of Engineering Education, 109*(3), 402–423. https://doi.org/10.1002/jee.20344
Flegg, J., Mallet, D., & Lupton, M. (2012). Students' perceptions of the relevance of mathematics in engineering. *International Journal of Mathematics Education in Science and Technology, 43*(6), 717–732. https://doi.org/10.1080/0020739X.2011.644333
FTS. (2022). *Teknologiutdanning 4.0: Anbefalinger for utvikling av NTNUs teknologistudier 2022-2030* [Technology education 4.0: Recommendations for development of NTNU's technology studies 2022-2030]. NTNU.
González-Martín, A. S., & Hernandes-Gomes, G. (2017). How are calculus notions being used in engineering? An example with integrals and bending moments. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 2073–2080). DCU Institute of Education and ERME.
Harris, D., Black, L., Hernandez-Martinez, P., Pepin, B., & Williams, J. (2015). Mathematics and its value for engineering students: What are the implications for teaching? *International Journal of Mathematical Education in Science and Technology, 46*(3), 321–336. https://doi.org/10.1080/0020739X.2014.979893
Langärds-Tarvoll, I. (2024). Understanding the challenges of the secondary-tertiary transition in mathematics for economics in higher education: A literature review. *Teaching Mathematics and its Applications: An International Journal of the IMA, 43*(4), 251–272. https://doi.org/10.1093/teamat/hrad011
Langärds-Tarvoll, I., & Göller, D. (2024). Bridging mathematical and microeconomic perspectives: A praxeological analysis of the Lagrange multiplier method. *Teaching Mathematics and its Applications: An International Journal of the IMA, 43*(4), 315–338. https://doi.org/10.1093/teamat/hrae020
Loch, B., & Lamborn, J. (2016). How to make mathematics relevant to first-year engineering students: Perceptions of students on student-produced resources. *International Journal of Mathematical Education in Science and Technology, 47*(1), 29–44. https://doi.org/10.1080/0020739X.2015.1044043
Lundheim, L. (2021). *Oscillatorar, tilbakekopling, difflikningar, dynamiske system og litt til* [Oscillators, feedback, differential equations, dynamical systems and some more]. Lecture notes for TTT4265 Electronic systems design and analysis I/II, Trondheim, NTNU.
Nilsson, J. W., & Riedel, S. W. (2011). *Electric circuits* (9th ed.). Pearson Education Inc.
Rønning, F. (2022). Learning mathematics in a context of electrical engineering. In R. Biehler, M. Liebendörfer, G. Gueudet, C. Rasmussen, & C. Winsløw (Eds.), *Practice-oriented research in tertiary mathematics education* (pp. 603–619). Springer.
Rønning, F. (2023a). *MARTA: Matematikk som redskap for tanken: Integrasjon av matematikk i ingeniørutdanning* [MARTA: Mathematics as a thinking tool: Integration of mathematics in engineering education]. NTNU.
Rønning, F. (2023b). Mathematics and engineering: Interplay between praxeologies. In T. Dreyfus, A. S. González-Martín, J. Monaghan, & P. W. Thompson (Eds.), *The Learning and Teaching of Calculus Across Disciplines – Proceedings of the Second Calculus Conference* (pp. 165–168). MatRIC. https://matriccalcconf2.sciencesconf.org/
Scanlan, J. O. (1985). The role of mathematics in engineering education: An engineer's view. *International Journal of Mathematical Education in Science and Technology, 16*(3), 445-451.