NTNU SEED

Center for Science & Engineering
Education Development

# Matematisk modellering

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# Student beliefs before…

**There is a single right way to solve every problem**

Assumptions are bad

If you are not on the "right track" you are lost

***"We are wasting half of the time to figure out what we are supposed to do"***

Mathematics is about following rules

Failure is… failure

**Focus on what the teacher wants**

If you cannot solve a problem, you need to collect more information.

*(we need to "unlock" the students!)*

# Student attitudes to problem solving

*"We always thought that there were ready-made formulas for everything."*

*"We had no structure in our problem solving. We googled a lot and mainly looked for shortcuts."*

*"The distinct difference between reality and mathematics was something we had never reflected over."*

*"We have only on very rare occasions been asked to solve an ill–defined problem."*

...

(2016 reports - mostly software engineering students end of year 2)

# Hvordan utvikle studentenes evne til å bruke matematisk modellering

- Mange studenter sliter med å bruke sine matematikkunnskaper i andre emner
- Hvordan lærer vi egentlig bort matematisk problemløsing?
- Hva er matematisk tenkning?

# Hva trenger en for å løse et problem?

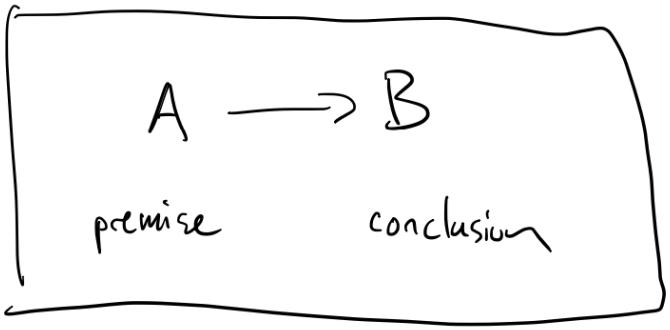
Huvudsakligt tillnärmande till problemlösning i skolan

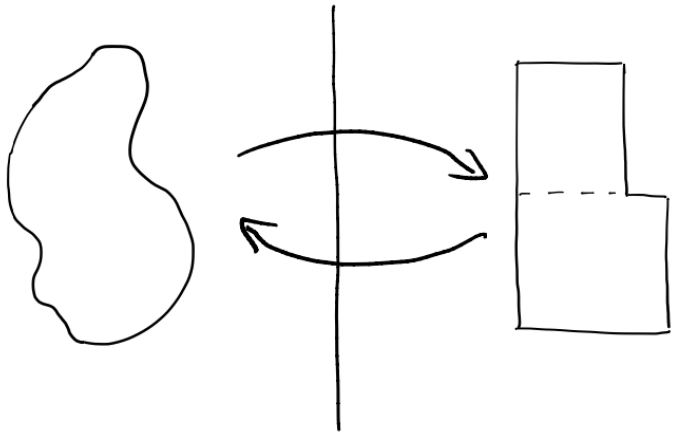
| | | | | |
|---|---|---|---|---|
| additional knowledge needed for solving a problem | = | knowledge from others | + | knowledge created by own thinking |

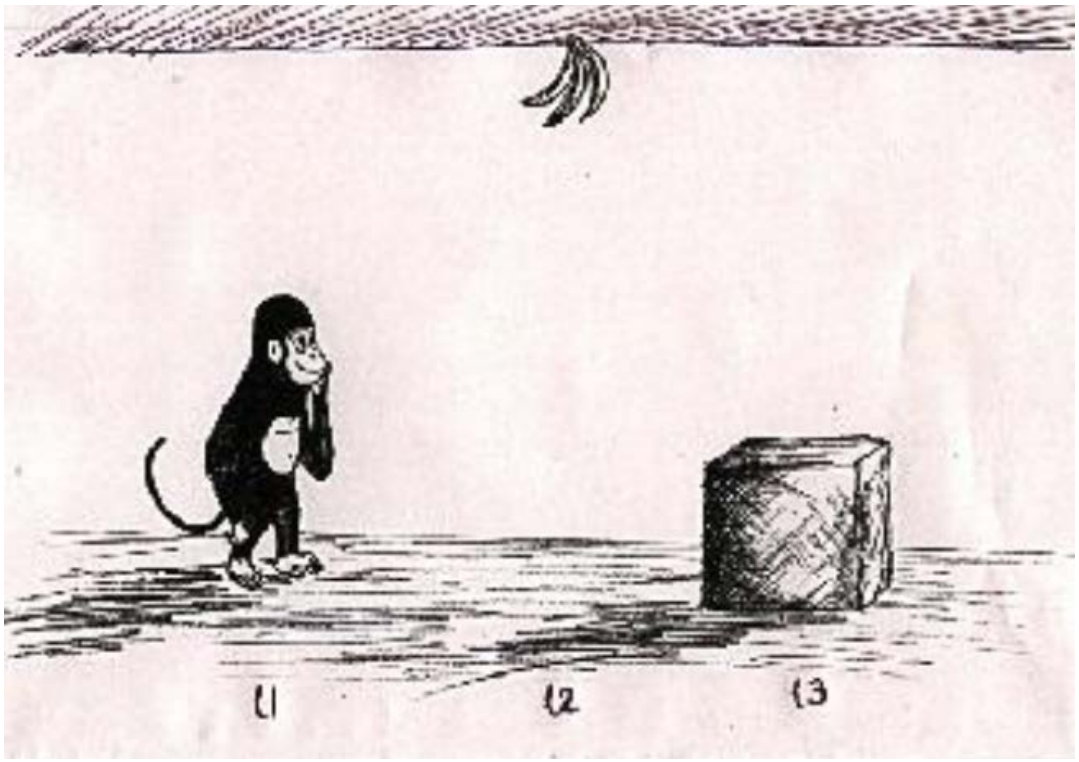
Genom att istället fokusera på den andra termen måste eget tänkande utvecklas

| | | | | |
|---|---|---|---|---|
| additional knowledge needed for solving a problem | = | ~~knowledge from others~~ | + | knowledge created by own thinking |

# Reasoning, modelling and problem solving





# Carefully selected problems

**simplified but realistic**

**highly varied**

**challenging**

can be remembered
as cases

only require familiar
background knowledge

…

# A tailor made learning environment

Cognitive apprenticeship

Case-based learning

Inquiry-based learning

(Work with Tom Adawi)

*Supervise by asking questions and by giving general problem solving advice in context!*

*Discuss mathematical thinking in general*

Follow-up and reflection in weekly modules

Adapt assessment to allow for creativity

**Teach students to investigate and to trust their own thinking!**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# Students learn quickly

(mid-course comments)

*"more mindful in making claims"*

*"more qualitative reasoning and understanding"*

*"more focus on process than answers"*

*"awareness of what you know/have/don't know"*

*"many possible answers"*

*"start with examples and then generalize"*

*"understand the problem"*

*"try things out"*

*"go back and revise"*

*"ask questions to yourself!"*

*"work in small steps"*

*"begin with the simple"*

*"assumptions allow you to continue"*

*"be more observant"*

# At the end of the course…

*"Kursen har även utvecklat mitt kreativa tänkande vad gäller matematiken, som tidigare bara varit mycket exakt och logisk för mig."*

***"We have learned a new way of thinking."***

*"The first course that made us feel as engineers."*

*"Math was so much more than just doing calculations."*

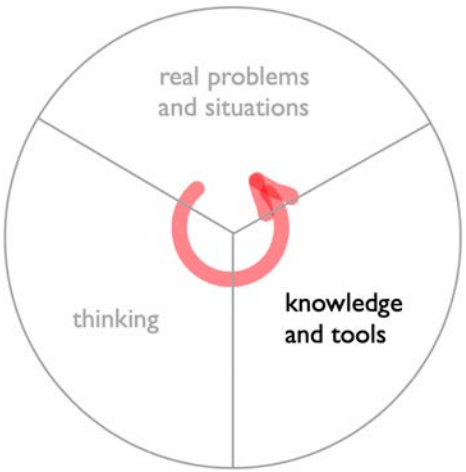
*"My rate of development has been enormous"*

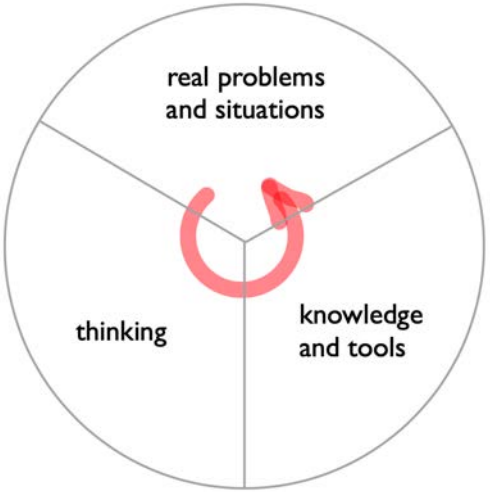
*"We have been able to solve much more difficult problems than we ever thought we could handle"*

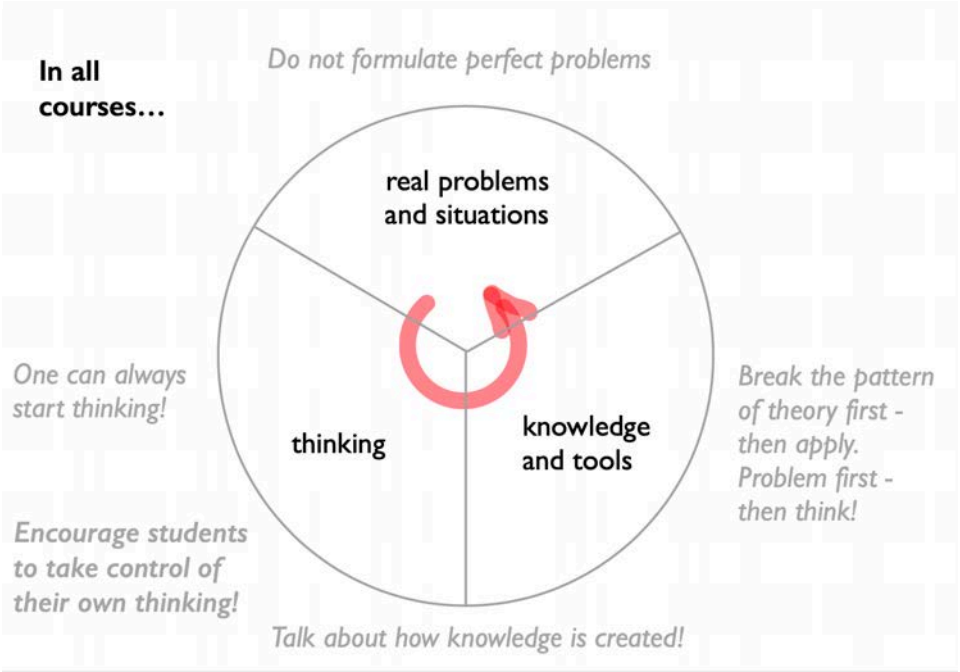
*"Imagine if we had been given more of this earlier in our education."*

…

# Problemdesign i alle emner…



# Examples and problems

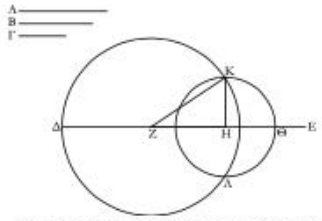
Rotary encoder
Lunch problem
Emergency care problem
Simple assignment
Translation problems
Consumer test ranking
Simple forecast
Arithmetic and geometric mean
Temperature control
Facility location
Balancing chemical reactions
Curve fitting
Map colouring
What is the revenue?
Shortest path
Achilles and the tortoise
Bridge problem
Beam on two supports
Predict weather
Medical test
Square root algorithm
Consumption problems
Reading everyday texts
When is optimality guaranteed?
Size of the world
Twelve balls problem
Random text (and music)
Renewable energy system
Bokeh
Homing
Estimation
Sound intensity
Throw ball
Project planning
Bouncing balls
Explain units
Whales and krill
Language recognition
Interpreting quantitative information
Expert system
Radioactive decay
Basic discrete structures
Medicine dose
Traffic simulation
Prove algebraic laws
Dice simulation
Data calibration
Computer graphics
Sorting complexity

ΣΤΟΙΧΕΙΩΝ α'. ELEMENTS BOOK 1

ΓΕΒ μείζων ἐστὶ τῆς ὑπὸ ΒΑΓ. ἀλλὰ τῆς ὑπὸ ΓΕΒ μείζων ἐδείχθη ἡ ὑπὸ ΒΔΓ· πολλῷ ἄρα ἡ ὑπὸ ΒΔΓ μείζων ἐστὶ τῆς ὑπὸ ΒΑΓ.

Ἐὰν ἄρα τριγώνου ἐπὶ μιᾶς τῶν πλευρῶν ἀπὸ τῶν περάτων δύο εὐθεῖαι ἐντὸς συσταθῶσιν, αἱ συσταθεῖσαι τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν ἐλάττονες μέν εἰσιν, μείζονα δὲ γωνίαν περιέχουσιν· ὅπερ ἔδει δεῖξαι.

Ἐκ τριῶν εὐθειῶν, αἱ εἰσιν ἴσαι τρισὶ ταῖς δοθείσαις [εὐθείαις], τρίγωνον συστήσασθαι· δεῖ δὴ τὰς δύο τῆς λοιπῆς μείζονας εἶναι πάντῃ μεταλαμβανομένας [διὰ τὸ καὶ παντὸς τριγώνου τὰς δύο πλευρὰς τῆς λοιπῆς μείζονας εἶναι πάντῃ μεταλαμβανομένας].




Ἔστωσαν αἱ δοθεῖσαι τρεῖς εὐθεῖαι αἱ Α, Β, Γ, ὧν αἱ δύο τῆς λοιπῆς μείζονες ἔστωσαν πάντῃ μεταλαμβανόμεναι, αἱ μὲν Α, Β τῆς Γ, αἱ δὲ Α, Γ τῆς Β, καὶ ἔτι αἱ Β, Γ τῆς Α· δεῖ δὴ ἐκ τῶν ἴσων ταῖς Α, Β, Γ τρίγωνον συστήσασθαι.

Ἐκκείσθω τις εὐθεῖα ἡ ΔΕ πεπερασμένη μὲν κατὰ τὸ Δ σημεῖον δὲ κατὰ τὸ Ε, καὶ κείσθω τῇ μὲν Α ἴση ἡ ΔΖ, τῇ δὲ Β ἴση ἡ ΖΗ, τῇ δὲ Γ ἴση ἡ ΗΘ· καὶ κέντρῳ μὲν τῷ Ζ, διαστήματι δὲ τῷ ΖΔ κύκλος γεγράφθω ὁ ΔΚΛ· πάλιν κέντρῳ μὲν τῷ Η, διαστήματι δὲ τῷ ΗΘ κύκλος γεγράφθω ὁ ΚΛΘ, καὶ ἐπεζεύχθωσαν αἱ ΚΖ, ΚΗ· λέγω, ὅτι ἐκ τριῶν εὐθειῶν τῶν ἴσων ταῖς Α, Β, Γ τρίγωνον συνέσταται τὸ ΚΖΗ.

Ἐπεὶ γὰρ τὸ Ζ σημεῖον κέντρον ἐστὶ τοῦ ΔΚΛ κύκλου, ἴση ἐστὶν ἡ ΖΔ τῇ ΖΚ· ἀλλὰ ἡ ΖΔ τῇ Α ἐστιν ἴση. καὶ ἡ

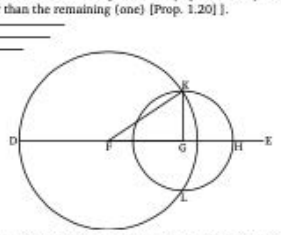
(the sum of) $BD$ and $DC$.

Again, since in any triangle the external angle is greater than the internal and opposite (angles) [Prop. 1.16], in triangle $CDE$ the external angle $BDC$ is thus greater than $CED$. Accordingly, for the same (reason), the external angle $CEB$ of the triangle $ABE$ is also greater than $BAC$. But, $BDC$ was shown (to be) greater than $CEB$. Thus, $BDC$ is much greater than $BAC$.

Thus, if two internal straight-lines are constructed on one of the sides of a triangle, from its ends, the constructed (straight-lines) are less than the two remaining sides of the triangle, but encompass a greater angle. (Which is) the very thing it was required to show.

## Proposition 22

To construct a triangle from three straight-lines which are equal to three given [straight-lines]. It is necessary for (the sum of) two (of the straight-lines) taken together in any (possible way) to be greater than the remaining (one), [on account of the (fact that) in any triangle (the sum of) two sides taken together in any (possible way) is greater than the remaining (one) [Prop. 1.20]].




Let $A$, $B$, and $C$ be the three given straight-lines, of which let (the sum of) two taken together in any (possible way) be greater than the remaining (one). (Thus), (the sum of) $A$ and $B$ (is greater) than $C$, (the sum of) $A$ and $C$ than $B$, and also (the sum of) $B$ and $C$ than $A$. So it is required to construct a triangle from (straight-lines) equal to $A$, $B$, and $C$.

Let some straight-line $DE$ be set out, terminated at $D$, and infinite in the direction of $E$. And let $DF$ made equal to $A$, and $FG$ equal to $B$, and $GH$ equal to $C$ [Prop. 1.3]. And let the circle $DKL$ have been drawn with center $F$ and radius $FD$. Again, let the circle $KLH$ have been drawn with center $G$ and radius $GH$. And let $KF$ and $KG$ have been joined. I say that the triangle $KFG$ has

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**Thinking creates knowledge!**



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# Noen resursser

- Maththinking.one
- Matmod.one
- Artikkel: «**Teaching Mathematical Modelling and Problem Solving - A Cognitive Apprenticeship Approach to Mathematics and Engineering Education**» *Dag Wedelin, Tom Adawi, iJEP,* Vol 4, No 5 (2014). http://dx.doi.org/10.3991/ijep.v4i5.3555

# Spørsmål

- Vad hoppas du att studenterna skall lära sig på det här viset som de inte annars får möjlighet till?
- Vad ser du som den största pedagogiska utmaningen för att etablera en kurs av det här slaget?
- Var hämtar du inspiration till uppgifterna?
- Kan du berätta något om bakgrunden till att du skapade den här kursen?

- Finns det några studieadministrativa utmaningar att tänka på?