Reasoning and proving in primary education (ProPrimEd)

0. Relevance to the call

This project seeks to strengthen the knowledge base for teachers' professional practice and teacher education through a solid collaboration between a university-based research group, the field of practice and international researchers. The project's objective is to investigate and support students' learning and teachers' practices in primary school, and pre-service teachers' learning in teacher education, within reasoning and proving in mathematics.

A new mathematics curriculum is being devised in Norway and is scheduled to be implemented by 2020. The curriculum will be organised around some core aspects in mathematics, one of which is *reasoning and justification*. The definition of this core aspect in the curriculum corresponds with the notion of reasoning and proving (RP) as proposed by Stylianides (2008), who uses it to denote activities involved in the process of identifying patterns, making conjectures about relations between mathematical objects involved, and providing arguments for validity of the conjectures. The new curriculum will employ a stronger emphasis on RP than the previous one (from 2006), as RP is set to get a prominent place in the mathematics curriculum. Our focus on the proposed project concerns primary-school mathematics, as the need for more knowledge about students' learning and teachers' practices within RP work in primary-school mathematics is emphasised in several research reviews. This project addresses thematic priority Area A in the call—students' learning processes—in addition to Area B—teachers' practices—as students' opportunities to learn are greatly determined by the teaching practices employed. Moreover, this project addresses opportunities in teacher education for developing pre-service teachers' (PSTs) practices within RP in primary-school mathematics.

The users of this project's results and outputs will be primary-school mathematics teachers and teacher educators, and, at the next stage, primary-school students. The participating teachers will be heavily involved in the project through a binding collaboration with schools and close work with three teachers from each school. Representatives from the schools are also members of the project's steering committee. The project will further contribute to strengthening teacher educators' work for preparing PSTs to teach RP in primary school.

1. Excellence

1.1 State of the art, knowledge needs and project objectives

The nature of mathematics as a discipline is characterised by investigating, reasoning with and proving different properties of mathematical objects and the relations between them. Although some differences on epistemological views exist (e.g., Balacheff, 2008; Reid & Knipping, 2010), researchers in mathematics education agree that mathematical proof is a core issue in the challenging task of learning and teaching mathematics, and that reasoning and proving are vital for learning school mathematics at all grade levels and in all topics. Proof is often perceived as having a formal and rigorous nature, heavily based on algebraic expressions. In the proposed project, we follow a broader definition of proof offered by Stylianides (2007) and developed for mathematics education. The definition respects mathematics as a discipline, as the modes of reasoning are valid mathematically. Simultaneously, the definition emphasises the community's role: A proof can be expressed in forms other than algebraic symbols, depending on the community's previous knowledge. The proposed project is motivated by needs raised by this upcoming new Norwegian curriculum, as well as by a lack of knowledge in primary-school RP in mathematics education research, as outlined below.

The need for research in lower-grades. While research in mathematics education has provided a considerable body of knowledge concerning RP at the secondary-school level (mostly in geometry) and in higher education, knowledge about RP at the primary-school level is limited. In their comprehensive literature review, Stylianides, Stylianides and Weber (2017) show that only a minority of reviewed extant studies addressed issues concerning learning argumentation and proof in school settings, other than secondary school and in domains other than Euclidean geometry. A quick glance at recent specialised volumes on proof and proving confirms the persistence of this tradition. For instance, in the latest specialised volume on proof and proving in mathematics education (Stylianides & Harel, 2018), only two out of 16 principal chapters focus on primary school.

The need for intervention studies. Several researchers (e.g., Stylianides & Stylianides, 2013) have called for intervention studies that explore what tasks to use when teaching RP and how teachers can support students' engagement in their work on RP. One critical aspect of intervention studies is how well they can be scaled up and become relevant to others besides the research community, such as teachers who are not part of research (Cobb, Jackson, & Dunlap, 2016). Stylianides and Stylianides (2013) argue that short time interventions are easier to scale up, as it is more practical to incorporate them into existing teaching and curriculum structures. Another way to address the problem of scaling up is to acknowledge teachers' current practices and take these as starting points for designing interventions (Cobb, Jackson, Smith, Sorum, & Henrick, 2013). Thus, to support learning and teaching RP in primary school, a need exists to develop interventions with short durations that acknowledge teachers' current practices as a starting point.

The need for more research in teacher education. To prepare new teachers to do the kind of teaching emphasised in research on RP in school mathematics and highlighted by the upcoming new Norwegian curriculum, this project addresses opportunities in teacher education for developing student teachers' practices within RP in primary-school mathematics. A growing knowledge base exists in mathematics-education research concerning the different aspects of PSTs' work on RP, e.g., their beliefs about proving, their modes of reasoning and their knowledge (e.g., Stylianides, Stylianides, & Philippou, 2007; Winsløw & Durand-Guerrier, 2007; Enge & Valenta, 2015). However, we do not know much about the process of PSTs' learning how to teach RP during teacher education and how teacher education can support their learning across course work and the practice field.

Project objective. The project's overall objective is to **develop innovative knowledge on students' learning, teachers' practices and pre-service teachers' learning-to-teach within work on reasoning and proving, in order to design teaching resources for primary education that can support that work**. This is recognised as important both in literature and in the upcoming new Norwegian curriculum. RP is to be integrated into all work on mathematics to support students' learning, but it can be reasonable to address some particular aspects, such as what it means to reason and prove in mathematics, and why we do it, in a sequence of lessons emphasising critical moments. The developed teaching resources will promote teachers, PSTs and students' learning (i.e., educative curriculum material, see Davies & Krajcik, 2005), and consist of:

a) A sequence of lessons that emphasise important aspects of RP in primary school mathematics. We assume that the sequence will comprise four to five mathematics lessons. The material will include tasks, directions for the teacher and examples of students' work and teachers' actions.
b) A description of teaching practices that help teachers (or PSTs) incorporate RP into all mathematics teaching to support students' learning in mathematics.

The resources will be theory-driven, empirically tested and developed through an intervention at partner school S1 in a close collaboration with teachers from the school and in close relation to their usual teaching. The resources will be developed further to increase their amenability to scaling up through a study of their use and implementation in another school, partner school S2. In addition, we will study how the knowledge developed through investigations in primary schools can be implemented in teacher education.

We establish four secondary objectives, listed below, that will lead to the achievement of the primary objective. We elaborate further on the secondary objectives in the next sections.

A. We will study the state of the art by documenting primary mathematics teachers' current instructional practices and students' opportunities for learning RP in primary school.
B. We will develop theory-driven and empirically tested resources that effectively can support mathematics teachers' practices and students' opportunities for learning RP in primary school.
C. We will investigate how resources developed in B can contribute to discursive changes in primary-school classrooms. Based on that, we will further develop the resources to make them more amenable for use in primary school.

D. Building on knowledge developed in A and B, we will study approaches in teacher education that can support pre-service teachers' learning-to-teach RP in primary school across learning contexts in teacher education—across course work and the practice field.

The collaboration with the S1 and S2 schools will take place in third through seventh grades. The project's results and outcomes in the next stage will contribute to the development of teaching practices during the earliest years (grades 1 and 2).

1.2 Ambition and novelty

As discussed above, a lack of knowledge exists in mathematics education research concerning RP in primary education. Extant research has shed light on different aspects of students and teachers' work on RP, including beliefs and misconceptions (especially at the secondary and tertiary levels), but more knowledge is needed about students' learning, teachers' teaching and PSTs' learning-to-teach and how these can be supported in primary education. This project will contribute to these aspects.

The resources for supporting RP work in primary-school mathematics will be developed through design-based research. The intervention will focus on amenability for scaling up, an aspect that many researchers have identified as difficult, but important. Further on, different approaches for working with RP in teacher education will be developed and investigated, which will strengthen teaching both in primary school and teacher education.

The proposed project involves also novelty concerning the theoretical approach. Students, teachers and PSTs' work on RP often is analysed through learning outcomes, while the processes of learning and learning-to-teach RP are this project's foci. We build on socio-cultural learning theory and consider mathematics to be a specific type of discourse in which reasoning and proving are essential, as elaborated on below.

1.3 Research questions and hypotheses, theoretical approach and methodology

Based on a socio-cultural learning theory, Sfard (2008) developed the *commognitive* framework particularly for analysing mathematics learning and teaching, which will be this project's overall theoretical perspective. Following Sfard (2008), we consider mathematics to be a specific type of discourse in which RP is essential, and mathematics learning and teaching are viewed as participation in a particular discourse. The framework will give us opportunities to analyse the modes of students' participation in a given discourse and community, rather than the modes of individual students' thinking and reasoning. Within a commognitive framework, *discourse* is a special type of communication within a specific community that becomes mathematical through that community's use of words, visual mediators, narratives and routines. Narratives and routines are important to our project, and we elaborate more on these aspects later. Within a mathematical discourse, any sequence of utterances, spoken or written, that describes objects' properties or the relationships between objects is called a *narrative*. Mathematical narratives can be numerical, e.g., '½ is equivalent to 2/4', or more general, e.g., 'Addition is commutative'. Narratives are subject to endorsement or rejection, i.e., being labelled as true or false based on specific rules that the community establishes. Endorsement of narratives is mathematics discourse's principal goal, which includes the processes of constructing new endorse-able narratives, substantiating them and recalling them in new situations. In other words, narratives are statements about mathematical properties and relations that are to be reasoned with, justified and proved. The way this happens depends on routines in a given community. *Routines* are well-defined practices a given community regularly employs in a discourse. Sfard (2008) describes routines as patterns guided by two sets of rules: those telling the participants *how* to act and those indicating *when* to do the given actions (e.g., Lavie, Steiner & Sfard, 2018).

Our project focuses on participation in a mathematical discourse on RP, involving the processes of identifying patterns, making conjectures and providing arguments on whether the conjectures are true or false. Hence, from a discursive stance, we are primarily interested in routines associated with the construction and substantiation of narratives. Thus, learning to reason and prove in mathematics is about individualising both the *when* and *how* of construction and substantiation routines. In commognitive terms, teaching is defined as a communicational activity intended to bring learners' discourse closer to a canonical discourse (Tabach & Nahclieli, 2015). Applying a commognitive perspective, Jeannotte and Kieran (2017) developed a conceptual model of mathematical reasoning based on exhaustive analyses of mathematics education research in which they identify processes related to reasoning and proving. To obtain insight into how students learn and teachers teach about RP, it is useful to delineate the possible patterns of the processes and actions involved in constructing and substantiating narratives. The model that Jeannotte and Kieran (2017) developed provide a lens to use during analyses.

Commognition will be our overall theoretical perspective during the project. In addition, in work on secondary objectives C and D, we consider Wenger's (1998) social learning theory of communities of practice as useful in analysing learning in collectives (among teacher colleagues in school or among PSTs at university) and in interactions with teaching resources. A *community of practice* is understood here as a group of people who, sharing a common concerns or passions, come together to explore these concerns and ideas, and share and grow their practice. Following Wenger (1998), learning is about becoming a more central member of a community of practice, with its *mutual engagement, joint enterprise* and *shared repertoire*. This shared repertoire can be considered a *repertoire of resources* (Pepin, Gueudet, & Trouche, 2013), and in secondary objective C, we will investigate how in-service teachers adopt teaching resources. By situating knowledge and learning in mathematics teaching relative to communities of practice, we also can describe the content and structure of universities and partner schools' practices in which mathematics PSTs are engaged. These aspects are raised in secondary objective D.

Secondary objective A: We will study the state of the art by documenting primary mathematics teachers' current instructional practices and students' opportunities for learning RP. To meet this objective, we will conduct a study involving teachers and students at the two partner schools, S1 and S2. Within our chosen theoretical framework, learning is understood as change in discourse. Thus, documenting the state of the art in teaching RP can be thought of as describing the current RP discourse in the classroom. Distinguishing the teacher's role in this discourse can be done by focusing particularly on certain actions/practices that the teacher performs while participating in the discourse. As routines can be understood in a broader setting than that of a mathematical discourse, it makes sense to think of teachers' actions as teaching routines (Nachlieli & Tabach, 2018; see also Lavie, Steiner, & Sfard, 2018). This comprises the three research questions' theoretical setup, which systematises our work on secondary objective A.

- A1: During mathematical discourse in the classroom, which narratives are accepted without justification, which narratives are subject to argumentation and what routines can be identified in the last situation?
- A2: Which teaching routines can be identified within RP in primary-school mathematics?
- A3: What opportunities for students' RP learning can be identified in primary-school mathematics?

We will observe three mathematics classes from each of the two schools. Each class will be observed for a period of one to two weeks. All lessons during this period will be video-recorded. The six teachers participating in the study will be interviewed several times during the observation period to gather information about what they view as their goals when teaching, as well as their reflections after lessons. This provides additional data for question A2.

Secondary objective B: To meet secondary objective B, development of theory-driven and empirically tested resources that effectively can support mathematics teachers' practices and students' opportunities for learning RP, we will conduct a classroom-design study (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Gravemeijer & Cobb, 2006). Building on prior research (including findings from secondary objective A) and through collaborating with three teachers in school S1, we will develop, test and revise an initial design or a conjecture about means for supporting students' learning-to-reason-and-prove in primary-school mathematics (i.e., a conjectured local instruction theory, see Gravemeijer and Cobb (2006)). The design will focus on developing instructional tasks, classroom interactions and teachers' discursive actions (Kwon, Ju, Kim, Park, & Park, 2013). The design study will contain three cycles of (re)designing and testing instructional activities based on analysis of students' learning and teachers' discursive acts when participating in the instruction experiments. From the iterative process of invention and revision, the intended product of the study is educative curriculum materials that support both teachers and students' learning (Davis & Krajcik, 2005). The material will include a sequence of instructional activities and associated instructional resources, along with local instructional theory on RP that comprises the rationale for the instructional sequence. Secondary objective B will be met by answering the following questions:

- B1: What can be features of the content and form of tasks that can promote students' learning of RP in primary school mathematics?
- B2: What might be a promising instructional sequence to address crucial mathematical aspects of RP and effectively supporting students' learning?
- B3: What teaching routines can promote students' RP in primary-school mathematics?
- B4: What discursive changes (e.g., use of words, routines, etc.) can take place in the classroom when a local instruction theory of RP is developed and implemented?

The data from the design study will include video recordings from all classroom sessions, pre- and post-interviews with students, copies of students' work and field notes. In addition, we will make video recordings from the research group meetings (between researchers and teachers) to document interpretations, conjectures and decisions made in the design study.

Secondary objective C: To increase the amenability for scaling up the intervention, we will investigate how teachers who did not participate in the design of resources use resources developed in secondary objective B and how these resources can contribute to discursive changes in primary-school classrooms. In school S2, three teachers will implement the resources in their teaching. We will investigate discursive changes in their classrooms (Sfard, 2008), i.e., how teachers' use of resources can influence students' participation in RP discourse and, thus, their learning to reason and prove in primary-school mathematics. Remillard (2005) discusses the relation between intended resources and enactment of resources as a way of analysing teachers' interactions with resources. This will be part of our investigation. Viewing resources developed in B as possible joint resources within a community of primary mathematics teachers, we will investigate how teachers adopt the resources, due to their identification with and exercised ownership of the resources. Based on the knowledge gained, we will develop the resources further to make them more amenable for use in primary school. Thus, the intended product is a revised version of the resources developed in B. Secondary objective C will be met by answering the following research questions:

- C1: How do teachers engage and work with resources, and what are the relationships between intended and enacted resources?
- C2: What discursive changes can take place in the classroom when previously designed resources for teaching RP are implemented?
- C3: What characterises teachers' collective interactions with resources, i.e., how do teachers identify with and exercise ownership of resources?

The data for all three research questions will include video recordings from all classroom sessions, pre- and post-interviews with participating teachers, copies of students' work and field notes. In addition, we will make video recordings from possible teacher meetings before and after their classroom teaching to document their interpretations and decisions.

Secondary objective D: To support PSTs' learning to teach across course work and the practice field in teacher education, Grossman et al. (2009) suggest working on core practices/routines of teaching and emphasising three approaches that effectively support PSTs' learning: use of different representations of practices (e.g., video cases, observations and teaching resources); decomposition of practices to make components more visible and practice more learnable for PSTs; and use of approximations of practices in which the complexity of teaching is partly reduced so that PSTs can concentrate on some aspects of teaching. Several studies have shown that work on representation, decomposition and approximation of important practices in mathematics teaching is effective for PSTs' learning (e.g., Lampert et al., 2013; Tyminski et al., 2013; Sleep, 2012). Our work on secondary objective D will start with an investigation of the state of the art in teacher education concerning work on learning to teach RP in mathematics. Building on knowledge developed from secondary objectives A and B, we will develop representations, decompositions and approximation of practices vital for teaching RP in primary-school mathematics and investigate how the approaches can promote PSTs' learning-to-teach RP across different contexts—from course work to the practice field. Our research questions are:

- D1: What opportunities and constraints for PSTs' learning-to-teach RP exist when undergoing transitions between campus courses and school placement?
- D2: How can teaching routines that promote students' RP in mathematics be implemented in teacher education to support PSTs' learning-to-teach RP in mathematics?
- D3: How can PSTs' identification with and ownership of instructional activities in RP be fostered when implementing them through approximation of practices?

We will integrate knowledge and resources developed in A and B into the course Mathematics 2 in teacher education for grades 1-7, as RP is emphasised as a topic in that course's curriculum. We first will study the state of the art concerning work on RP, opportunities and constraints with PSTs' learning in and across course work, and school placement. In the next stage, we will develop and try out approaches for work with PSTs based on our work on A and B, and study how they affect PSTs' opportunities to learn to teach RP. The data will comprise video recordings of PSTs during whole-class course work and focus-group interviews/post-session reflections with PSTs.

2. Impact

2.1 Proposed project's potential impact

The proposed project will make a significant impact both on societal and scientific levels. On the societal level, it will strengthen the quality of mathematics education in primary school and mathematics teacher education in Norway, and in the long run, it will strengthen the quality of mathematics education internationally. The knowledge and resources developed from work on secondary objectives A, B and C are important for integrating RP into school mathematics, which is part of the upcoming mathematics curriculum in Norwegian primary schools and is emphasised in research studies in mathematics education. By doing classroom-design-based research in close collaboration with teachers and using interventions of short duration and close to teachers' existing practice, we will ensure that the resources developed from the project will be adaptable to other mathematics teachers.

Furthermore, the project's secondary objective D will contribute to necessary knowledge on how teacher education can strengthen PSTs' opportunities to learn how to teach RP in mathematics. We will gain knowledge about how different communities of practice in teacher education can provide effective contexts for PSTs' learning-to-teach, a question raised by several educators (e.g., Peressini et al. 2004). At the end of the project, we will publish an open-access textbook for use in Norwegian teacher education, thereby ensuring that the outcomes are available to relevant users.

In the fourth UN sustainable-development goal, numeracy is highlighted as a particularly desired education outcome (target 4.6). Extant literature on numeracy shows that RP is closely connected to numeracy skills (e.g., Kilpatrick, Swafford & Findell, 2001); thus, this project is linked to the UN's sustainable-development goals. In addition, the project will contribute to target 4.1, which emphasises equitable and quality primary education that leads to relevant and effective learning outcomes. Furthermore, STEM education, including mathematics education, is highlighted by UNESCO, which emphasises the need to raise young people's awareness of mathematical concepts, as well as attract them to mathematics-related studies and careers. By supporting students' learning in RP, which is a fundamental feature of mathematics, the project aims to make students better informed and prepared for making decisions concerning their future studies and careers.

The project's research questions and expected outcomes will make a solid scientific impact on mathematics education research in the international community. The proposed project will be visible in the international mathematics education research community through our participation at well-established conferences and direct communication with research groups. This project will contribute to filling a research gap concerning RP in primary education as well. An innovative theoretical framework will be used to analyse RP learning and teaching that will generate new insights in this research area. Furthermore, the project will contribute to more knowledge on scaling up intervention studies. The project also will make a local impact through substantial capacity building by engaging young researchers at NTNU in the project, collaborating with international researchers and employing two PhDs and one post.doc.

2.2 Measures for communication and exploitation

The resources developed in the project and summaries of the research conducted will be published on the project's website and presented at national conferences, seminars and meetings of mathematics teachers, as described in the dissemination plan. The teachers participating in the project will contribute to presentations on their experiences using the resources in teaching. They also will arrange workshops for other teachers, particularly during mathematics teachers' annual national conferences (November Conference and LAMIS). The teachers also will be involved in writing popular scientific papers in *Tangenten*, a Norwegian journal for mathematics teachers. Another user group is mathematics teacher educators. Throughout the project period, we will present results at an annual conference for Norwegian mathematics teacher educators. At the end of the project, an open-access book for use in mathematics-teacher education will be published. More details are provided in the dissemination plan.

On the scientific level, the project group's researchers will disseminate the project's outcomes through papers in reputable journals. Every research question will elicit at least one high-quality scientific paper. As presented in the dissemination plan, the project group's researchers will attend one or more influential international conferences annually during the project period, presenting results connected to the research questions. This also will result in publications in conference proceedings, some of which are level 1 publishing channels. It is anticipated that some of the scientific writings will entail joint work with members of international research groups that collaborate on the project.

3. Implementation

3.1 Project manager and project group

Project organisation
Associate Professor Anita Valenta will lead the proposed project and take overall responsibility for work on all secondary objectives. She has participated in several projects concerning mathematics teacher education and was a project leader for the 'Number Sense' project, which aimed to develop resources for use in teacher education and professional development courses. Besides Valenta, the research group will comprise (in alphabetical order) Associate Professor Kristin Krogh Arnesen, Associate Professor Ole Enge, PhD student Sigrid Iversen and Associate Professor Kirsti Rø. In addition, visiting professor at NTNU Andreas Stylianides will participate in the research group. Arnesen, Enge, Rø and Valenta all have experience with and competence within research and development in mathematics (teacher) education, which holds relevance for the proposed project, as shown in their CVs. They will use all their research time (45% of full time) on the project during the project period and work on development, data collection, research and dissemination. In the past 15 years, Professor Stylianides from the University of Cambridge has made great contributions in the area of reasoning and proving in mathematics. He will do research with other project members and provide feedback and advice on drafts of papers. The project group also includes two PhD students and one post-doc. The PhDs will be employed for 4 ears, with 25% teaching duties. Iversen, one of the PhD students, already is employed and will start working on her project 1 August 2019 with funding from NTNU, independent of this application to NFR. The project also will facilitate research projects for 10 to 12 master's students who will have access to data from the project to conduct small-scale, deep-focus studies, such as students' modes of reasoning in a given type of task, classroom talk/analysis, etc.

The research group has established collaborations with two primary schools, and three teachers from each school will participate in the project (possibly only two teachers at school 2). At this stage, the schools are unable to specify which teachers will participate in the project, but agreements on participation have been reached with representatives from each school: Erik Amundsen and Jonas Larsen. In addition, a binding collaboration has been established with Trondheim Municipality.

International partnerships and collaborations
In addition to the collaboration with Professor Stylianides from the University of Cambridge, UK, who is a member of the project group, the project proposal includes collaborations with two international research groups, one from Italy and one from Sweden (see also 3.2).

Italy has a tradition in educational research on the subject of argumentation and proof in mathematics. We will collaborate with a group comprising four researchers: Professor Pier Luigi Ferrari, who is interested in research on the role of linguistic competence in students' written argumentations; teacher-researcher Rossella Garuti, who is experienced in work on argumentation and mathematical proof in primary and lower secondary schools; researcher Francesca Martignone, who has been working on analysis of anticipation processes during exploration activities in open problems; and Associate Professor Cristina Sabena, who has been working on design-based research aimed at intertwining argumentation with formative assessment.

Linnæus University in Sweden has a long tradition of using socio-cultural learning theories in research on mathematics teaching and learning, including Wenger's (1998) theory on communities of practice. Therefore, an interest in teaching mathematics as collective learning provides common ground for a collaboration. We will collaborate with two researchers: Associate Professor Hanna Palmer, who has been working on primary-school teachers' professional identity development, teaching and learning in primary school and task design, and Helen Sterner, a teacher educator/PhD student who is working on teacher-focussed design research based on mathematical reasoning and generalisation in primary-school algebra.

3.2 Project organisation and management

The project will run from 1 August 2020 to 31 July 2024 and utilise a steering committee that will meet regularly to discuss development and realisation of the project to ensure that it is relevant for primary schools, as well as provide advice about dissemination and relevant forums. The steering committee will comprise representatives from authorities and participating schools and two researchers: Anita Valenta, Kirsti Rø, Ingeborg Ranøyen (Trondheim Municipality), Erik Amundsen (school 1) and Jonas Larsen (school 2). Valenta will lead the steering committee.

The project group, led by Valenta, will be responsible for overall project management and oversee data collection, analysis and interpretation of results. The work will be organised in work packages, and each package will have a designated leader chosen on the basis of experience and competence, as elaborated on below. The project group will meet regularly and work closely on project development and research. Even though all members will participate in discussions of important aspects of the project, duties on different work-package analysis and research will be organised in smaller groups for practical reasons. As described earlier, the teachers will contribute both in development and implementation of resources and in dissemination of the project's results, particularly communication with teachers. NTNU's Department of Teacher Education and the faculty's research-support section will provide financial aid and day-to-day project operations.

As for collaborations with researchers from Italy and Sweden, we plan to conduct annual workshops, with one collaborating group at a time. During these workshops, we intend to share results, ideas and future plans on the aforementioned common-ground topics and possibly also do research together and lay groundwork for more formal research collaborations.

Work packages
To address the project's objectives, four work packages—1, 2, 3 and 4—have been devised, corresponding specifically to the four secondary objectives—A, B, C and D—respectively. The relation between the work packages and the resources to be developed in the project is outlined in Figure 1.

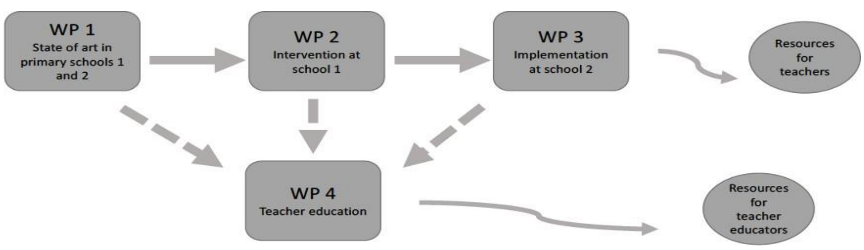

Figure 1: Relation between work packages and development of resources

The secondary objectives were described earlier, but here, we provide some additional, mostly practical information. An overview of work in the packages and plans for when it will be done is presented in Figure 2 below. (Please see the dissemination plan for further details; the progress plan encompassing the project's principal activities and milestones is provided in the electronic grant application form.)

	Autumn 2020	Spring 2021	Autumn 2021	Spring 2022	Autumn 2022	Spring 2023	Autumn 2023	Spring 2024
WP 1 -state of art in primary schools	Data collection at school 1 and 2	Transcription of data, analysis	Analysis of data, writing papers, dissemination					
WP2 -intervention at school 1		Intervention at school 1 and data collection	Transcription of data, analysis	Analysis of data, writing papers, dissemination				
WP 3 -implementation at school 2				Implementation at school 2 and data collection	Transcription of data, analysis	Analysis of data, writing papers, dissemination Work on the final version of resources for primary school		
WP 4 -teacher education		State of art -data collection	Transcription of data, analysis	Intervention at teacher education and data collection	Transcription of data, analysis	Analysis of data, writing papers, dissemination		

The first version of the resources completed

PhD 1 completes the thesis

The final version of the resources for teachers completed

Open access book for teacher educators completed
PhD 2 completes the thesis

Figure 2: Overview of work in the packages and main milestones

Work package 1 corresponds to secondary objective A, a study of the state of the art concerning RP in primary school. Arnesen will lead the work, but all project members will be involved to some degree, as the findings are the basis for the other work packages.

Work package 2 is related to secondary objective B, an intervention at partner school S1 and development of resources in close collaboration with teachers from partner school S1. Enge has had a collaboration with one primary school teacher over a period of six years, observing classroom teaching and closely working with the teacher on development of instruction. This experience is valuable for secondary objective B, and Enge will lead work in package 2. Iversen (PhD student 1) has extensive experience as a teacher, and her PhD project will be part of secondary objective B and work package 2. Project members working on work package 3 also will be involved as the insight in approach and findings from work package 2 are important for package 4.

Work package 3 corresponds to secondary objective C and addresses implementation and further development of resources developed in B. In her PhD project, Rø considered learning as identity development (Rø, 2018), and her competence in the area is valuable for secondary objective C. Therefore, Rø will lead work package 3. A post-doc with experience and competence on research teachers' use of resources will be engaged to work on this package.

Work package 4 corresponds to secondary objective D and addresses implementation of knowledge and resources developed in A and B in teacher education. Valenta will lead this part, as her work on 'Mastering Ambitious Mathematics Teaching', a project aimed at designing a professional development course and resources based on an approach through approximation of practice suggested by Lampert et al. (2013), is particularly valuable for work on secondary objective D. PhD student 2's PhD project will comprise part of work in this package.

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