

Question 1 (30 points)

Define and explain the following concepts:

a. Lognormal distribution

A stochastic variable is lognormally distributed if its log is normally distributed. This distribution is useful for describing gross financial returns, partly because a lognormal variable always is positive, and partly because a normally distributed instantaneous return implies a lognormally distributed annual (discrete period) return. The lognormal distribution is skewed to the left. Its expectation is a little greater than the exponentiated value of the expectation of the underlying normal variable:

$$\ln X \sim N(\mu, \sigma^2) \Rightarrow EX = e^{\mu + \sigma^2/2} > e^\mu$$

b. Dynamic programming and Euler equation

Dynamic programming is a term used to characterize decision making in a setting where the implications of the decisions accrue over time, but only current variables can be decided currently. Decisions made according to dynamic programming determine current variables under the assumption that future decisions will be made in the same manner. The Euler equation is the first-order condition for current consumption. It describes the optimal tradeoff, in expectation, between current and future consumption, with financial returns playing the role of relative price. It shows that if, in equilibrium, the agent receives an additional \$1, this agent will be indifferent between investing it and spending it on current consumption.

c. Relative risk aversion, elasticity of intertemporal substitution, and the relationship (if any) between the two

The coefficient of relative risk aversion is defined as the elasticity of marginal utility,

$$\gamma = -\frac{u'' c}{u'}$$

It describes the agent's aversion to unknown changes in consumption. The elasticity of intertemporal substitution describes the elasticity of the ratio of expected future to current consumption with respect to its relative price, i.e. the expected rate of return. Its reciprocal describes the agent's aversion to known or planned changes. With expected CRRA utility, this reciprocal equals the coefficient of relative risk aversion, meaning that the agent is equally averse to known and unknown variations. Epstein-Zin preferences allow them to be different.

d. Certainty equivalence

The certainty equivalent of a random outcome is the outcome that, if known to occur with certainty, would give the same utility as the expected utility of the random outcome. The difference between expected return and the certainty equivalent is referred to as the risk premium.

e. Collateral as used in credit markets

This term is used about assets belonging to a borrower that the lender pledges as security for the loan, such that the lender may seize them in case the borrower fails to service the loan. In a situation of asymmetric information, lenders typically demand collateral as a safeguard against the possibility that the borrower runs off with the borrowed money instead of investing it as agreed as condition for the loan.

Question 2 (30 points): Securitization

a. Outline the model by Gennaioli, Shleifer, and Vishny.

Households are infinitely risk averse and want to invest their endowment in secure (AAA) assets. Intermediaries are professional investors, risk neutral, and face two kinds of investment objects/projects: High-quality projects, no uncertainty, and high return, in limited supply; and low-quality projects, with uncertain return. The uncertainty is both idiosyncratic and aggregate. Securitization allows the intermediaries to trade portfolios of projects with each other. By reducing idiosyncratic risk, securitization enables intermediaries to offer households larger amounts of investments/deposits at returns that are promised to be riskless.

b. Explain and discuss the tradeoff between microeconomic efficiency and macroeconomic fragility in their model.

Securitization spreads the idiosyncratic risks among the various intermediaries. Full securitization eliminates the idiosyncratic risk completely. This is microeconomically efficient because it enables households to get positive returns on larger deposits. However, because securitization erases the difference between lucky and unlucky intermediaries, it makes all intermediaries equally exposed to aggregate risks. With full securitization, no intermediary fares better than others in case the worst aggregate scenario (recession) is realized.

c. Explain and discuss the importance of neglected aggregate risk in this model.

As long as all risks are fully understood and taken into account, equilibrium in this model gives the best possible outcome given the aggregate scenario that is realized. However, if the risk of the worst scenario (recession) is ignored, intermediaries will advertise unrealistically favorable terms to households. The microeconomic efficiency remains; and as

long as no recession occurs, the outcome is good. However, if a recession occurs even though the risk of it had been ignored, some intermediary firms will go under. If securitization has gone far enough, even some idiosyncratically fortunate intermediaries will go under. If securitization is complete, all intermediaries will fail.

Question 3 (40 points): The Equity Premium Puzzle

Outline, discuss, and compare the relative merits of the following two models:

- a. Rare disasters
- b. Habit formation

The two respective models should be presented by their key equations. The key assumptions should be explained verbally.

In equilibrium, either model predicts an equity premium in excess of what is implied by the standard model. The equilibrium solutions should be presented and interpreted. Additional credit for deriving them.

The rare-disasters model claims that fat tails make risk greater than what is implied by the variance of consumption/dividend growth in the standard model with normal or lognormal distributions. The habit-formation model looks instead at the nature of risk aversion and claims that habit formation makes risk aversion variable. In particular, it is countercyclical in the sense that agents become more risk averse when equity returns are low and vice versa. On average, risk aversion will be greater than in the standard model. This result depends on a particular specification of how habits are updated in response to changes in actual consumption.