Exam FIN 3005 Asset pricing – December 8, 2021

1. **Exercise 1**

a) Empirical phenomenon. Risk adjusted stock markets returns, e.g. measured by the Sharpe ratio are higher than reasonable risk aversion assumptions and consumption data should imply. The puzzle was established based on US post WW II data, but our textbook claims it holds for most countries. From economic theory one can derive the mathematical relationship

$$\frac{E[R^m] - R^F}{\sigma(R^m)} \leq \gamma\sigma(\Delta),$$

where the left hand side is the Sharpe-ratio of the market return, γ the relative risk aversion coefficient, and $\sigma(\Delta)$, the standard deviation of consumption. From US data (different sources report slightly different numbers, the above numbers are from our textbook) the risk premium $E[R^m]-R^F$ is 8%, market volatility $\sigma(R^m) = 16\%$, so that the Sharpe-ratio is 0.5. The standard deviation of consumption is 1% (again, different sources report different numbers, but they are not very different). These numbers imply that the coefficient should be 50. It is believed that reasonable values of γ are lower than 10, possibly around 2.

b) Something is wrong with the theory or the data. High values of γ (≥ 50) lead to an interest rate puzzle; the risk free interest rate becomes extremely sensitive for small consumption shocks. Possible sources of errors: Extremely high return in all markets last 70 years (just a lucky outcome). Hard to measure true standard deviation of consumption, the standard deviation is typically based on annual data which therefore are smoothed over the year. The model is based on power utility, this function may be too simple to model preferences.

c) In class we did a quick-and-dirty estimation of the equity puzzle, based on Norwegian data, instantly collected from various sources on the internet. It showed that, whereas the risk premium is about the same in Norway as in the US, the market volatility is higher. If we assume that the risk premium is the same 8% (we got a slightly lower number in class) and the market volatility is 22%, the Sharpe ratio is 0.36. Based on the US value of the standard deviation of consumption of 1%, the γ value is 36. We also estimated the standard deviation of consumption from consumption data from SSB and found a value higher than 2%. With 2%, $\gamma = 18$, which still is a high value. So the equity premium puzzle seems to exist also in Norway (the reservation is only due to the sloppy nature of our in-class research), but not to the same extent as in the US. The explanation for this is the higher market volatility and consumption standard deviation in Norway than in the US.

2. **Exercise 2**

a) Compare expected utility in the cases with and without insurance.
Without insurance:

$$U = u(10) + \frac{9}{10}u(10) + \frac{1}{10}u(5) = -0.21.$$

With insurance of cost 1:

$$U = u(9) + u(10) = -0.2111.$$

Here,

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

and $\gamma = 2$.
Expected utility is higher in the case without insurance than in the case with insurance. The agent would not buy insurance.

b) The consumption loss is $X = 5$ in the case of disaster.

$$E[X] = \frac{9}{10} \cdot 0 + \frac{1}{10} \cdot 5 = 0.5.$$

c) Let π be the maximum price the agent is willing to pay. We determine π so that the agent is equally well of, i.e. has the same expected utility with and without insurance.

$$u(10-\pi) + u(10) = u(10) + \frac{9}{10}u(10) + \frac{1}{10}u(5) = -0.21.$$

So, we need to find π so that

$$u(10-\pi) + u(10) = -0.21.$$

This equation can be solved in closed form. The solution is $\pi = \frac{10}{11} = 0.909$. Observe that the agent is willing to pay more for the insurance than the expected loss. Also, the insurance price in a) is higher than π.

3. **Exercise 3**

Consider a one period model with two time points, time 0 and time 1. Assume that three states are posible at time 1, called states 1, 2, and 3, respectively. Three securities, called A, B, and C, may be traded at time 0 for prices (in NOK) $A_0 = \frac{5}{9}$, $B_0 = \frac{1}{3}$, and $C_0 = \frac{14}{9}$, respectively. The securities have the following strictly positive time 1 payoffs: Security A pays 1 NOK in state 1, and 1 NOK in state 3. Security B pays 1 NOK in state 2, and 1 NOK in state 3. Security C pays 2 NOK in state 1, and 1 NOK in state 2.

a) The payoffs of the securities A, B, and C can be replicated by portfolios of state price securities. Sec A:

$$\pi_1 + \pi_3 = A_0 = \frac{5}{9}$$

Sec B:

$$\pi_2 + \pi_3 = B_0 = \frac{1}{3}$$

Sec C:

$$2\pi_1 + \pi_2 = C_0 = \frac{14}{9}$$

We have three equation with three unknowns state prices π_1, π_2, and π_3. The solution is $\pi_1 = 16/27$, $\pi_2 = 10/27$, and $\pi_3 = -1/27$. A negative state price means that you would require compensation today for receiving one unit in that state. What is the interpretation of that? You might think of this as a situation where the state price in state 3 really is zero ($1/27 \approx 0,037$), but there is an externality connected to state 3 that makes it even more unattractive.

b)

$$R^f = \frac{1}{\sum_i \pi_i} = \frac{1}{16/27 + 10/27 - 1/27} = 27/25 = 1.08.$$

c)

$$S_0 = \pi_1 + 2\pi_2 + 3\pi_3 = 1\frac{2}{9} \approx 1.2222.$$

d) Option payoff in state 1:

$$\max[K - S(1), 0] = \max[3 - 1, 0] = 2.$$

Option payoff in state 2:

$$\max[K - S(2), 0] = \max[3 - 2, 0] = 1.$$

Option payoff in state 3:

$$\max[K - S(3), 0] = \max[3 - 3, 0] = 0.$$

This is exactly the same payoff as security C, so this option must therefore have the same price as security C which is $\frac{14}{9} \approx 1.5556$.

4. **Exercise 4**

a) Main idea: The agents well-being are determined by consumption relative to habits. Utility is therefore only assigned to consumption above habits. Habits are external, determined by aggregate consumption. By construction the habits are slow moving and always less than consumption. This mechanism replaces the standard constant relative risk aversion coefficient with a time dependent function η_t in the equity puzzle equation (see Exercise 1),

$$\eta_t = \frac{\gamma}{S_t},$$

where

$$S_t = \frac{C_t - X_t}{C_t},$$

is the time t surplus consumption ratio, X_t the time t habit, and C_t the time t consumption. In good times when S_t is high, η_t is low, and the market Sharpe ratio low due to low returns/high prices. In bad times when S_t is low, η_t is high, and the market Sharpe ratio high due to high returns/low prices.

b) Utility function is

$$u(C_t, X_t) = \frac{(C_t - X_T)^{1-\gamma}}{1-\gamma},$$

where C_t and X_t are time t consumption and habit, respectively. The marginal utility can conveniently be expressed by the surplus consumption ratio as

$$u'(C_t, X_t) = (S_t C_t)^{-\gamma}.$$

The stochastic discount factor is

$$m_{t+1} = \beta \frac{u'(C_{t+1}, X_{t+1})}{u'(C_t, X_t)} = \beta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{\gamma},$$

where β is the subjective (gross) disount rate.

c) The risk free interest rate puzzle occurs in the standard model. Small consumption shocks lead to large changes in the risk free interest rate. By construction in the Campbell-Cochrane model, the interest rate is constant (independent of consumption shocks) , which roughly corresponds to observed risk free interest rates. Therefore, there is no risk free interest rate puzzle in their model.