① Example (8 points)

(20 point)

	Studies the material (C)	Doesn't study the material (D)
Passes exam (A)	650	50
Doesn't pass exam (B)	20	280

	S (C)	S^c (D)	Σ
(A) P	0.65	0.05	0.7
(B) P^c	0.02	0.28	0.3
Σ	0.67	0.33	0.1

1. Conditional prob: The prob. of an event such that it is determined by the existence of another event. (2 points)

Ex: What's the probability that a student passes a final exam of a course given that the student never studies that course's material? $P(P|S^c) = \frac{P(P \cap S^c)}{P(S^c)} = \frac{0.05}{0.33} = 0.15$ (2 points)

2. Joint prob.: The prob. for two events to occur simultaneously. (2 points)

Ex: What's the probability that a student passes a final exam of a course and doesn't study that course's material? $P(P \cap S^c) = 0.05$ (2 points)

3. Addition law: The prob. that either one or both events occur. (2 pts)

Ex: What's the probability that a student either passes the exam or studies the material? $P(P \cup S) = P(P) + P(S) - P(P \cap S)$

$= 0.7 + 0.67 - 0.65 = 0.72$. (2 points)

(2)

a. Testing the committee's statement is to challenge an assumption. In this case, the null hypothesis (H_0) is the assumption to be challenged. (4 points)

$$H_0: \mu \leq 300$$
$$H_a: \mu > 300$$

b. We cannot conclude that the committee's statement is wrong. (1 point)

c. We can conclude that mean travel bill for the conference is more than \$300. (1 point)

d. The Type I error is rejecting H_0 when it's true. If we commit this error, this would mean that the mean travel bill is concluded to be more than \$300 when it is in fact at most \$300. (2 points)

e. The Type II error is accepting H_0 when it's false. If we commit this error, this would mean that the mean travel bill is concluded to be at most \$300 when it is in fact more than \$300. (2 points)

② (Cont'd)

f. Since the population standard deviation is known, we can calculate the test statistic z.

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{309 - 300}{30/\sqrt{100}} = 3.00$$

$P(z \geq 3.00) = 1 - 0.9987 = 0.0013 < \alpha = 0.01$

⇒ We conclude that we find sufficient statistical evidence to reject H_0 at the 0.01 level of significance. (5 points)

g. Using the standard normal probability table, we find that $z = 2.33$ provides an area of 0.01 in the upper tail ⇒ $z_{\alpha=0.01} = 2.33$.

Since test statistic $z > z_\alpha$ ⇒ We reject H_0. (5 points)

③ (10 points)

Let p_1 = population proportion of UiO's job finding rate among students that have not handed in their thesis.

p_2 = population proportion of NTNU's job finding rate among students that have not handed in their thesis.

a. $H_0: p_1 - p_2 = 0$ (2 points)

$H_a: p_1 - p_2 \neq 0$

b. $\bar{p}_1 = \frac{600}{1200} = 0.50$

$\bar{p}_2 = \frac{700}{1500} = 0.47$ (2 points)

c. $\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{600 + 700}{1200 + 1500} = 0.48$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.5 - 0.47}{\sqrt{0.48(0.52)\left(\frac{1}{1200} + \frac{1}{1500}\right)}} = 1.55$$

For the test statistic $z = 1.55$, a two-tailed test

p-value is $2(1 - 0.9394) = \underline{0.1212} > \alpha = 0.05$.

We cannot reject the H_0. We conclude that the job-finding rates do not differ between the two universities. (5 points)

d. $z_{\alpha/2} = z_{0.025} = 1.96 >$ test statistic $z = 1.55$. Cannot reject H_0. (1 point).

(4) $0.3^2 = 0.09$

a. $H_0: \sigma^2 = 0.09$ (2 points)

$H_a: \sigma^2 \neq 0.09$

b. We know $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

Since $\begin{cases} n = 101 \\ s^2 = 0.4^2 = 0.16 \end{cases} \Rightarrow \frac{100(0.16)}{\chi^2_{0.025,100}} \leq \sigma^2 \leq \frac{100(0.16)}{\chi^2_{0.975,100}}$

$$\Rightarrow \frac{16}{129.561} \leq \sigma^2 \leq \frac{16}{74.222}$$

$0.12 \leq \sigma^2 \leq 0.22$ (5 points)

c. Test statistic $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{100(0.16)}{0.09} = 177.78$

$D_f = 101 - 1 = 100$

The test statistic χ^2 provides an area smaller than 0.005 in the upper tail of the chi-square distribution.

$\Rightarrow$ The p-value is less than $2(0.005) = 0.01 < \alpha = 0.05$

We reject H_0 and conclude that the variance in the equity market return in the first quarter of 2022 is different from it in 2021. (5 points)

d. $H_0: \sigma^2 \leq 0.09$

(3 points)

$H_a: \sigma^2 > 0.09 \Rightarrow$ Research hypothesis ($\Rightarrow H_a$)

e. $\left.\begin{array}{l} \chi^2 = 177.78 \text{ (from part c)} \\ \chi^2_{\alpha, n-1} = \chi^2_{0.05, 100} = 124.342 \end{array}\right\} \chi^2 > \chi^2_{\alpha, n-1}$

$\Rightarrow$ Reject H_0. We can conclude that the variance of the equity market return in the first quarter of 2022 is larger than in 2021. (5 points)

(5)

a) $H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

Since firm A has a larger sample variance $\Rightarrow$ refer to firm A as population 1.

$F = \dfrac{s_1^2}{s_2^2} = \dfrac{18000}{12000} = 1.5$ with $n_1 - 1 = 60$ numerator df
$n_2 - 1 = 40$ denominator df.

$\underset{\text{0.1 area in upper tail}}{1.47} < \underset{\text{test stat.}}{1.5} < \underset{\text{0.05 area in upper tail}}{1.64}$

$\Rightarrow 0.2 > p\text{-value} > 0.1$ (2-tailed test)

Since $\alpha = 0.1 \Rightarrow$ we cannot reject H_0. (5 points)

(The critical value at $\alpha = 0.1$ is $F_{0.1/2} = F_{0.05}$ in each tail $= 1.64$.

Test statistic $F < F_{\alpha/2} \Rightarrow$ cannot reject H_0.)

b) Since Firm C has a larger sample variance $\Rightarrow$ pop. 1

$H_0: \sigma_1^2 \leq \sigma_2^2$

$H_a: \sigma_1^2 > \sigma_2^2$ (research hypothesis)

$$F = \frac{s_1^2}{s_2^2} = \frac{13920}{12000} = 1.16 \quad \text{with } 60 \text{ num. df}, \; 40 \text{ den. df}.$$

$1.47 > 1.16$

0.1 area in the upper tail

test statistic

The area in the upper tail must be greater than 0.1

$\Rightarrow$ p-value $> \alpha = 0.1$

$\Rightarrow$ cannot reject H_0.

(The critical value is $F_{0.1} = 1.47 >$ F test statistic $= 1.16$

$\Rightarrow$ cannot reject H_0)

(5 points)

MC questions require calculations.

1) $P(F) = 0.45$; $n = 6$

$$P(X=0) = \binom{6}{0} 0.45^0 \, 0.55^6 = 1(1)(0.0277) = \boxed{0.0277}$$

2) P(at least 2 microcomputers)

$= 0.3 + 0.2 + 0.2 = \boxed{0.7}$

3) With every 1-minute interval being equally likely, the assembly time random variable is :

$$f(x) = \begin{cases} 1/30 = \boxed{0.033} & , \text{ for } 160 \leq x \leq 190 \\ 0 & , \text{ otherwise} \end{cases}$$

4) $P(z < 1.16) = \boxed{0.877}$

5) $\mu = 90$, $\sigma =$

$$P(80 \leq X \leq 95) = P\left(\frac{80-90}{12} \leq z \leq \frac{95-90}{12}\right)$$

$$= P(-0.83 \leq z \leq 0.42) = 0.6628 - 0.2033 = 0.4595 = \boxed{45.95\%}$$

6) $\mu = 40000$, $\sigma = 5000$

$$P(X \geq 30000) = P\left(z \geq \frac{30000-40000}{5000}\right) = P(z \geq -2) = \boxed{0.9772}$$