

Exam SØK3524 Spring 2022

Question 1 (60%)

Two firms, 1 and 2, are polluting a lake. The Environmental Protection Agency (EPA) has decided to cap the emissions to \bar{M} per unit of time. Without abatement (business as usual) the emission will be \hat{M}_i ($i = 1, 2$). Due to different technology the abatement cost of the two firms is different. The cost functions are given as $C_i = a_i(\hat{M}_i - M_i) + b_i(\hat{M}_i - M_i)^2$ where $(\hat{M}_i - M_i)$ is the amount of abatement.

- a) Formulate the EPA's planning problem when the goal is to minimise total abatement cost.
- b) Find the social optimal emission of the two firms and demonstrate the conditions for $M_i \geq 0$.
- c) Demonstrate how both a tax on emission and a cap-and-trade regime can generate the socially optimal solution.
- d) Consider the situation when the EPA cannot observe the emission from each of the firms. The concentration of pollution in the lake can, however, be observed and is given by $E = d_1 M_1 + d_2 M_2$ where d_i are the "transportation" coefficients. The damage function reads $D = \alpha E$. Find the social planner solution of pollution E^* , emission and abatement of the two firms when the goal is to minimize total cost. The firms are facing the following tax scheme $T_i = t(E - E^*)$. Propose a tax rate t .
- e) Analyze the problem in d) under the assumption of the increasing marginal damage function $D = \alpha E + \frac{\beta}{2} E^2$.
- f) Discuss taxes and quota regulation in light of dynamic efficiency and information uncertainty.

Question 2 (40%)

Consider a fishery where the utility of fishing only depends on the harvest volume. h_t is the harvest at time t and $U(h_t)$ is the utility, where $U'(h_t) > 0$ and $U''(h_t) < 0$. The stock's dynamics is governed by $\dot{X}_t = F(X_t) - h_t$, where $F(X_t) = rX_t(1 - \frac{X_t}{K})$ is the natural growth of the stock.

- a) Formulate the problem and find the conditions for optimality when the aim is to maximise the utility of the fishery from today until $T = \infty$.
- b) Use phase diagrams to analyse the dynamics of the fishery.