Eksamen 1101 svarantydning 0615

**SØK 1101 Environmental and resource Economics, spring 2015.**

**Question 1**

a) We have the logistic function $F(X) = rX(1 - X / K)$ . It is easily illustrated graphically with it's zero points $X = 0$ and $X = K$ . Without fishing we have (*) $dX / dt = rX(1 - X / K)$ which indicates a stable stock for $X = K$ when starting from $X_0 > 0$ . The carrying capacity $K$ is therefore the highest stock Mother Nature can carry in the long-term when there is no fishing. The interpretation of the intrinsic growth rate is best seen when transforming (*) to $(dX_t / dt) / X_t = r(1 - X_t / K)$ where the left hand side indicates the instanous growth rate. The growth rate of the fish stock is therefore a linear function of the density $X$ . The highest growth rate is equal to $r$ , called the intrinsic growth rate, or maximum specified growth rate, when the stock is close to zero.

b) With fishing we have $dX / dt = F(X) - h = rX(1 - X / K) - h$ and where we that find the stock increases if natural growth dominates fishing, and *vice versa*. Something about this relationship can for example be said when $h = \bar{h}$ is fixed over time, and when a fixed fraction of the stock is harvested, $h = \alpha X$ . In the first case we typically have two equilibria, in the second case one equilibrium. We have an equilibrium fishery when $dX / dt = F(X) - h = rX(1 - X / K) - h = 0$ , or $h = rX(1 - X / K)$ . $X_{msy}$ is found when $F'(X) = 0$ which yields $X_{msy} = K / 2$ . We then also find $h_{msy} = F(X_{msy}) = rK / 4$ .

c) $h = qEX$ is the Schaefer catch function and the interpretation should be straightforward. When considering an equilibrium fishery we have $qEX = rX(1 - X / K)$ . Solving for the stock size (when $X > 0$ ) we find $X = K(1 - qE / r)$ . Therefore, the stock is a negative linear function of the effort use. This can be shown in a figure. Inserted into the catch function we next find $h = qEX = qEK(1 - qE / r)$ which give the harvest as a logistic-type function of the effort use (number of boats, or number of nets, etc.). With the revenue as $ph$ and the cost as $cE$ , we then have the profit as a function of the effort use as (**) $\pi = pqEK(1 - qE / r) - cE$ . A figure should be drawn to show the social efficient solution where the profit is at its highest; that is, the largest distance between the income $pqEK(1 - qE / r)$ schedule and the linear cost function $cE$ . This is the maximum economic yield solution, $E_{mey}$ . The solution should be related to $E_{msy}$ . It will of course strengthen the answer if the analytical expression for $E_{mey}$ is found by optimizing (**). When next inserted into $X = K(1 - qE_{mey} / r)$ , $X_{mey}$ can be found, and when inserting into $h = qE_{mey}K(1 - qE_{mey} / r)$ the optimal harvest is found. The profit function

$\pi = pqEK(1-qE/r)-cE$ could also be depicted in marginal terms as marginal revenue (MR) and marginal costs (MC).

d) Open access is a situations with no regulations and *de facto* no property rights. Somewehat imprecisely this is 'the tragedy of the commons'. It is defined by zero profit, $\pi = pqEK(1-qE/r)-cE = 0$. This solution should be shown in a figure where also the social efficient solution is shown. Analytical expressions for $E_\infty$ etc. should also be shown. Important observations are $E_\infty > E_{mey}$ and $X_\infty < X_{mey}$. We usually expect $E_\infty > E_{msy}$, but the opposite may happen if, say, the harvesting price $c$ is 'high'. High $c$ and low $p$ always curb fishing.

**Question 2**

a) Should be straightforward. Important difference between the short-term and long-term pro arguments.

b) Also straightforward. The stock pollution can be exemplified by the climate problem, and illustrated by the growth equation $dS/dt = E - G(A)$ with $E$ as the emission and $G(A)$ as the decay function (cleansing). The pollution stock $S$ therefore increases if the emission exceeds that of the natural decay. A pure flow pollution 'disappear' at the same moment as the pollution stops. Example: noise.

c) This problem was taken up in Exercise 2, and a model of this type could have been used. Important to mention how the quota ('the cap') initially is distributed. Typically either given away for free, or auctioned. Also something about the cost efficiency of the quota mechanism when the firm minimizes the sum of the abatement and quota costs while taking the quota price as given (no market power).

d) Make a distinction between renewable and conditionally renewable resources. The conceptual difference between conditionally renewable (e.g., a fish stock) and non-renewable resources (e.g., oil) should be shown by the stock – flow relationships, $dX/dt = F(X)-h$ and $dX/dt = -h$. For a non-renewable resource we therefore find (in continuous time) $X_t = X_0 - \int_0^t h_\tau d\tau$ .

e) With the growth function $Q(t) = 0.1t^2 - 0.005t^3$ , we find $Q'(t) = 0.2t - 0.015t^2 = t(0.2-0.015t)$ and $Q''(t) = 0.2 - 0.03t$. The shape of this function and the maximum value can then easily be found and illustrated with a figure. The average value is defined by $Q(t)/\text{t} = 0.1t - 0.005t^2$, with a maximum value given by $d(Q(t)/t)dt = (Q(t)/t)' = (0.1-0.010t) = 0$. It should also be mentioned that the maximum average value also means that $Q'(t) = Q(t)/t$ holds. This is easily recognized when looking at the graph of the growth function.

The economic problem when just looking at one stand of trees with no replanting (no opportunity cost of the logging value) , is defined by $\max_t PV = pQ(t)e^{-\delta t} - c$ with $p$ as the net-logging value (timber price minus logging cost), $c$ as the planting cost and $\delta$ as the discount rent. We find $dPV / dt = p[Q'(t)e^{-\delta t} - \delta Q(t)e^{-\delta t}] = pe^{-\delta t}[Q'(t) - \delta Q(t)] = 0$ , or $Q'(t) = \delta Q(t)$ . This equation defines the optimal logging time. Inserted for $Q(t)$ and solving a second order equation, the economic optimal logging time can explicitly be found. With $\delta > 0$ it is easily recognized that the economic optimal logging time will be located on the increasing part of the growth function and hence lower than the maximum value found above.