1

a) $\pi_T = \max(\max(S_T^1, S_T^2) - X, 0)$

b) $$\frac{\pi_t}{B_t} = E_t^Q\left[\frac{\max(\max(S_T^1, S_T^2) - X, 0)}{B_T}\right]$$

Let A_1^2 be the event where $S_T^1 > S_T^2$ and A_2^2 be the event $S_T^2 > S_T^1$. Also, let A_1 be the event $S_T^1 > X$ and A_2 the event $S_T^2 > X$.

We can write π_T as follows:

$$\pi_T = S_T^1 \cdot 1_{A_1^2 \cap A_1} + S_T^2 \cdot 1_{A_2^2 \cap A_2} - X \cdot 1_{A_1 \cup A_2}$$

Thus

$$\frac{\pi_t}{B_t} = E_t^Q\left[\frac{S_T^1}{B_T} \cdot 1_{A_1^2 \cap A_1}\right] + E_t^Q\left[\frac{S_T^2}{B_T} \cdot 1_{A_2^2 \cap A_2}\right] - X E_t^Q\left[\frac{1}{B_T} 1_{A_1 \cup A_2}\right]$$

$$\Longleftrightarrow$$

$$\underline{\pi_t = S_t^1 Q_{S_1}(A_1^2 \cap A_1) + S_t^2 Q_{S_2}(A_2^2 \cap A_2) - \frac{B_t}{B_T} Q(A_1 \cup A_2)}$$

c)

```
sum = 0
for (i=1; i ≤ Nsim, i++)
{
```

2 dim. vector of independent ε ~ N(0,1)

volatility vector $\sigma_1 = \begin{bmatrix} \sigma_1^1 \\ \sigma_1^2 \end{bmatrix}$

eps = rann (2,1)

$S_T^1 = S_t^1 \, e^{(r - \frac{1}{2}\|\sigma_1\|^2)(T-t) + \sigma_1^T eps \cdot \sqrt{T-t}}$

$S_T^2 = S_t^2 \, e^{(r - \frac{1}{2}\|\sigma_2\|^2)(T-t) + \sigma_2^T \cdot eps \cdot \sqrt{T-t}}$

use BS

$call_T = \max(S_T^1 - K, 0)$

$\Pi_T = \max(\max(S_T^1, S_T^2) - X, 0) + (call_t \cdot e^{r(T-t)} - call_T)$

sum = sum + Π_T

}

$\Pi_t = e^{-r(T-t)} \cdot sum / Nsim$.

2 $\quad u(c) = \ln c \quad , u'(c) = \frac{1}{c}$

The agent solves the problem

$$\max_{\{c_0, c(s)\}} \quad u(c_0) + \beta \sum_{s\in S} \pi(s)\, u(c(s))$$

s.t

$$c_0 + \sum_{s\in S} pc(s)\, c(s) = y_0 + \sum_{s\in S} pc(s)\, y(s)$$

We form the Lagrangian

$$\mathcal{L} = u(c_0) + \beta \sum_{s\in S} \pi(s)\, u(c(s)) - \lambda\Big(c_0 + \sum_{s\in S} pc(s)\, c(s) - y_0 - \sum_{s\in S} pc(s)\, y(s)\Big)$$

FOC

$$\frac{\partial \mathcal{L}}{\partial c_0} = \frac{1}{c_0} - \lambda = 0 \Leftrightarrow \lambda = \frac{1}{c_0}$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = \beta \pi_1 \frac{1}{c_1} - \lambda pc_1 = 0 \Leftrightarrow \beta\pi_1 \frac{1}{c_1} - \frac{1}{c_0} pc_1 = 0$$

$$\Leftrightarrow c_0 = \frac{pc_1}{\beta \pi_1} \cdot c_1 = \frac{m_1}{\beta} c_1$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = \beta \pi_2 \frac{1}{c_2} - \overbrace{\frac{1}{c_0}}^{\lambda} pc_2 = 0 \Leftrightarrow c_0 = \frac{pc_2}{\beta \pi_2} \cdot c_2$$

From this we get

$$\frac{pc_1}{\beta \pi_1} c_1 = \frac{pc_2}{\beta \pi_2} c_2 \Leftrightarrow \frac{c_1}{c_2} = \frac{pc_2/\pi_2}{pc_1/\pi_1} = \frac{m_2}{m_1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow c_0 + PC_1 \cdot c_1 + PC_2 \cdot c_2 - y_0 - PC_1 \cdot y_1 - PC_2 \cdot y_2 = 0$$

$$\Leftrightarrow$$

$$\frac{PC_2}{\beta \pi_2} c_2 + PC_1 \cdot \frac{PC_2/\pi_2}{PC_1/\pi_1} \cdot c_2 + PC_2 \cdot c_2 - y_0 - PC_1 y_1 - PC_2 y_2 = 0$$

$$\Leftrightarrow$$

$$c_2 \left(\frac{PC_2}{\beta \pi_2} + PC_2 \frac{\pi_1}{\pi_2} + PC_2 \right) = y_0 + PC_1 y_1 + PC_2 y_2$$

$$\Leftrightarrow$$

$$c_2 = \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{PC_2 \left(\frac{1}{\beta \pi_2} + \frac{\pi_1}{\pi_2} + 1 \right)}$$

$$= \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{m_2 \left(\frac{1}{\beta} + 1 \right)}$$

$$c_1 = \frac{m_2}{m_1} \cdot c_2 = \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{m_1 \left(\frac{1}{\beta} + 1 \right)}$$

$$c_0 = \frac{m_1}{\beta} c_1 = \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{1 + \beta}$$

↳ a) $$c_0 = \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{1 + \beta}$$

b) $$c_1 = \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{m_1 \left(\frac{1}{\beta} + 1 \right)}$$

c) $$c_2 = \frac{y_0 + PC_1 \cdot y_1 + PC_2 \cdot y_2}{m_2 \left(\frac{1}{\beta} + 1 \right)}$$