NTNU
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Department of Economics

Examination paper for FIN3005 Asset Prising/Makrofinans

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Examination date:	07.06.2018
Examination time (from-to):	4 hours (09.00 – 13.00)
Censorship date:	28.06 2018

Permitted examination support material: C /Flg formelsamling: Knut Sydsæter, Arne Strøm og Peter Berck (2006): Matematisk formelsamling for økonomer, 4utg. Gyldendal akademiske. Knut Sydsæter, Arne Strøm, og Peter Berck (2005): Economists' mathematical manual, Berlin.
Calculator: Casio fx-82ES PLUS, Casio fx-82EX Citizen SR-270x, SR-270X College or HP 30S.

Language:	English, Norwegian (bokmål og nynorsk)
Number of pages (front page excluded):	3
Number of pages enclosed:	0

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig □ **2-sidig** □

sort/hvit □ **farger** □

skal ha flervalgskjema □

Checked by:

Date Signature

Text in English

Answer all questions.

1. Define gross and net equity return and risk-free return and explain the difference between gross and net returns. Why does it make sense to assume that gross equity returns are log-normally distributed? And what does this say about the choice of method for estimating the equity premium?

Main points expected:

- Definition of gross and net return in discrete time.
- Show that, if the instantaneous net return is r, then the one-period gross return is e^r. This was covered in class and lecture notes.Thus, if the instantaneous net return is normal, it is natural to assume that the one-period gross return is log-normal. The non-negativity of log-normally distrbiuted gross returns is an implication, not a motivation.
- If gross returns are lognormal, the equity premium shold be estiamted as the arithmetic means of the algebraic returns. The reason is that these are the sample moments corresponding to the theoretical expectation terms. This point was covered as part of the in-class feedback on the Fall 2017 term paper.

Max score: 30 points

2. Present and discuss the Gertler-Kiyotaki model of systemic runs. Discuss in particular the role of the possible existence of two equilibria in this model. How does it differ from traditional analyses of bank runs?

Main points expected:

- The basic setup of the model, especially the differential efficiency of households and (shodow) banks in making investments.
- The logic behind the respective assumptions, especially the one about asymmetric information, which limits (shadow) banks' leverage.
- The possibility of two equilibria, one with and one without a run.
- Runs are systemic, not bank specific.
- Full score requires math as well as explanation, but understanding is more important than memorized formulae.

Max score: 30 points

3. Consider the preference ordering defined by the following iteratively defined value function V_t:

$$V_t(A_t) = \exp\left\{(1-\beta)\ln c_t + \frac{\beta}{1-\gamma}\ln\left[E_t\left(V_{t+1}(A_{t+1})\right)^{1-\gamma}\right]\right\},$$

where A_t is current wealth, c_t current consumption, E_t the expectations operator for expectations conditional on information available at time t, and β and γ fixed parameters.

The dynamic wealth constraint is

$$A_{t+1} = R_{p,t+1}(A_t - c_t),$$

where

$$R_{p,t+1} = \omega_t R_{e,t+1} + (1 - \omega_t) R_{f,t+1},$$

$R_{e,t+1}$ and $R_{f,t+1}$ are the gross equity and risk-free rates of return, respectively, and ω_t the share of the portfolio invested in equities.

Assume that risk-free assets are in zero net supply.

a. Is this a special case of Epstein-Zin preferences? If so, how?

Answer: The logarithmic formulation is the limiting case of a unitary elasticity of intertemporal substitution, i.e. a unitary elasticity of intertemporal substitution.

b. Use dynamic programming to derive optimal consumption as a share of wealth.

Answer: Proceeding in the usual way, and using the dynamic wealth constraint, we derive the first-order condition for consumption as

$$(1 - \beta)/c_t = \beta[E_t V_{t+1}(A_{t+1})^{1-\gamma}]^{-1} E_t V_{t+1}(A_{t+1})^{-\gamma} V'_{t+1}(A_{t+1}) R_{p,t+1}.$$

Because this is a special case of Epstein-Zin preferences (and i.i.d. returns), we know that we can use the conjecture $V_t(A_t) = \Phi A_t$. Using this and substituting from the dynamic wealth constraint, we find

$$(1 - \beta)/c_t = \beta\left[E_t \Phi^{1-\gamma} A_{t+1}^{1-\gamma}\right]^{-1} E_t \Phi^{-\gamma} A_{t+1}^{-\gamma} \Phi R_{p,t+1}$$

$$= \beta\left[E_t \Phi^{1-\gamma} R_{p,t+1}^{1-\gamma} (A_t - c_t)^{1-\gamma}\right]^{-1} E_t \Phi^{-\gamma} R_{p,t+1}^{-\gamma} \Phi (A_t - c_t)^{1-\gamma} R_{p,t+1}$$

$$= \beta/(A_t - c_t),$$

so that

$$(1 - \beta)(A_t - c_t) = \beta c_t.$$

Dividing both sides by c_t, we see that

$$\psi_t \equiv c_t/A_t = 1 - \beta.$$

c. Use this condition (for t as well as $t + 1$) to derive the Euler equation and the condition for optimal portfolio allocation.

1. *Answer:* Use this condition (for t as well as $t+1$) to derive the Euler equation

$$1 = E_t\left[\beta(c_{t+1}/c_t)^{-1}R_{p,t+1}\right].$$

The result in Question 2, together with the dynamic wealth constraint, implies

$$1 = \frac{c_t/A_t}{c_{t+1}/A_{t+1}} = \left(\frac{c_t}{c_{t+1}}\right)\frac{A_{t+1}}{A_t} = \left(\frac{c_t}{c_{t+1}}\right)\frac{R_{p,t+1}(A_t - c_t)}{A_t}$$

$$= \left(\frac{c_t}{c_{t+1}}\right)R_{p,t+1}(1-\psi_t) = \left(\frac{c_t}{c_{t+1}}\right)R_{p,t+1}\beta.$$

Because this must be true in all states of the world, it is also true in expectation. Thus, the Euler equation is

$$1 = E_t\left[\beta(c_{t+1}/c_t)^{-1}R_{p,t+1}\right].$$

Note: This question was given as Exercise 3 in the Fall 2017 version of the course, and the full solution was provided then.

Max score: 40 points.

Norsk tekst (bokmål)

Besvar alle oppgaver.

1. Definer brutto og netto aksjeavkastning og risikofri avkastning. Forklar forskjellen mellom brutto og netto avkastning. Hvorfor er det rimelig å anta at brutto aksjeavkastning er lognormalfordelt? Og hva betyr dette for valg av metode for å estimere aksjepremien?
2. Presenter og diskuter Gertler og Kiyotakis modell for “runs” i finanssystemet. Diskuter spesielt hva det innebærer at denne modellen kan ha to likevekter. Hvordan skiller modellen seg fra tradisjonelle analyser av bank-“runs”?
3. Formelen i neste linje beskriver en preferanseordning som er definert iterativt ved hjelp av verdifunksjonen V_t:

$$V_t(A_t) = \exp\left\{(1-\beta)\ln c_t + \frac{\beta}{1-\gamma}\ln\left[E_t\big(V_{t+1}(A_{t+1})\big)^{1-\gamma}\right]\right\},$$

der A_t er dagens formue, c_t dagens konsum, E_t forventningsoperatoren gitt informasjon på tidspunkt t, og β og γ er faste parametere.

Formuen utvikler seg over tid i henhold til betingelsen

$$A_{t+1} = R_{p,t+1}(A_t - c_t),$$

der

$$R_{p,t+1} = \omega_t R_{e,t+1} + (1 - \omega_t) R_{f,t+1},$$

$R_{e,t+1}$ og $R_{f,t+1}$ er henholdsvis brutto aksjeavkastning og risikofri avkastning og ω_t aksjeandelen i porteføljen.

Anta at netto tilbud av risikofrie investeringsobjekter er null.

a. Er dette et spesialtilfelle av Epstein-Zin-preferanser? I så fall hvordan?
b. Bruk dynamisk programmering til å utlede optimalt konsum som andel av formuen.
c. Bruk denne betingelsen (for både t og $t + 1$) til å utlede Euler-likninga og optimalitetsbetingelsen for porteføljesammensetningen.

Norsk tekst (nynorsk)

Svar på alle oppgåver.

1. Definer brutto og netto aksjeavkastning og risikofri avkastning. Forklar skilnaden mellom brutto og netto avkastning. Kvifor er det rimeleg å anta at brutto aksjeavkastning er lognormalfordelt? Og kva tyder dette for val av metode for å estimera aksjepremien?
2. Presenter og diskuter Gertler og Kiyotakis modell for "runs" i finanssystemet. Diskuter særskilt kva det inneber at denne modellen kan ha to likevekter. Korleis skil modellen seg frå tradisjonelle analysar av bank-"runs"?
3. Formelen i neste linje beskriv ei preferanseordning som er definert iterativt ved hjelp av verdifunksjonen V_t:

$$V_t(A_t) = \exp\left\{(1-\beta)\ln c_t + \frac{\beta}{1-\gamma}\ln\left[E_t\big(V_{t+1}(A_{t+1})\big)^{1-\gamma}\right]\right\},$$

der A_t er dagens formue, c_t dagens konsum, E_t forventningsoperatoren gitt informasjon på tidspunkt t, og β og γ er faste parameterar.

Formuen utviklar seg over tid ut frå vilkåret

$$A_{t+1} = R_{p,t+1}(A_t - c_t),$$

der

$$R_{p,t+1} = \omega_t R_{e,t+1} + (1-\omega_t)R_{f,t+1},$$

$R_{e,t+1}$ og $R_{f,t+1}$ er høvesvis brutto aksjeavkastning og risikofri avkastning, og ω_t aksjedelen i porteføljen.

Anta at netto tilbod av risikofrie investeringsobjekt er null.

a. Er dette eit spesialtilfelle av Epstein-Zin-preferansar? I så fall korleis?
b. Bruk dynamisk programmering til å utleia optimalt konsum som del av formuen.
c. Bruk dette resultatet (for både t og $t+1$) til å utleia Euler-likninga og optimalitetsvilkåret for porteføljesamansetjinga.