FIN 3005
Fall 2019

Final exam
Answer key

Answer all questions. Their relative weights are indicated as points out of 100.

Question 1 (30 points)

Define risk aversion and intertemporal substitution and explain how the two concepts relate to each other. Outline how the rate of relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS) can be specified in different models of investor behavior.

Risk aversion refers to intolerance of wide variations of outcomes across states of the world. Intertemporal substitution refers to the willingness to tolerate planned or anticipated variations over time. In the power expected utility model of investor preferences, the elasticity of intertemporal substitution (EIS) is the reciprocal of the rate of relative risk aversion. The reason is that this specification is additive in two dimensions, over time as well as across states of the world. Epstein-Zin preferences allow the two to be separated. An investor whose rate of relative risk aversion is greater than the reciprocal of the EIS prefers early resolution of uncertainty and vice versa.

Question 2 (30 points)

Kiyotaki and Moore analyze how the use of capital as collateral for credit can exacerbate business cycle movements. Outline and explain the main points in their analysis and present your own critique of their model as a description of a part of the real economy.

Because credit is essential in this model, it needs to have two types of agents: impatient ones (farmers) who want to borrow and patient ones (gatherers) who want to lend. Furthermore, the farmer's presence must be essential to their production so that their output fails to materialize if they run away with the borrowed money. A recession lowers collateral values as well as investment, which in turn lowers future asset values. As future declines are anticipated already in the current period, they can greatly exacerbate business cycle movements of asset prices. Because asset prices are affected, a recession also affects the allocation of capital between high-productivity borrowers (farmers) and low-productivity lenders (gatherers). Thus, the presence of this financial friction makes business cycles more pronounced as well as longer lasting.

Question 3 (40 points)

Suppose log of the gross equity return can be approximated by the following linear equation:

$$r_{e,t+1} = -0{,}05 + 0.95 \ln P_{t+1} + 0.05 \ln D_{t+1} - \ln P_t,$$

where P_t is the price of equity and D_t the amount of dividends in period t.

Suppose further that consumption equals dividends and that its log gross growth rate develops over time according to:

$$g_{t+1} = x_t + \eta_{t+1},$$

where η_{t+1} is a normally distributed zero-mean white noise and x_t a long-term trend that moves according to

$$x_t = 0.025 + x_{t-1} + \varepsilon_t.$$

Here, ε_t is a normally distributed zero-mean white noise that is uncorrelated with η_t. The standard deviations of η_t and ε_t are $\sigma_\eta = 0.1$ and $\sigma_\varepsilon = 0.005$, respectively.

Suppose also that investors in this model have Epstein-Zin preferences and that the log of the stochastic discount factor is

$$m_{t+1} = 0.36 + 4g_{t+1} - 9r_{e,t+1}.$$

Finally, suppose the log of the price-dividend ratio can be approximated as the following linear relationship:

$$z_t \equiv \ln P_t - \ln D_t = 2 + ax_t.$$

a. Describe briefly the kind of puzzle this model is intended to solve.
b. Derive the value that the parameter a must have for this model to be internally consistent. Explain the nature of the constraint that you must use for this purpose.
c. Compute the equity premium in this model.
d. Compute the rate of relative risk aversion and the elasticity of intertemporal substitution. [Hint: use the formula for the discount factor with Epstein-Zin preferences:

$$M_{t+1} = \beta^{(1-\gamma)/(1-\delta)}(c_{t+1}/c_t)^{-\delta(1-\gamma)/(1-\delta)}\mathrm{R}_{p,t+1}^{(\delta-\gamma)/(1-\delta)}.]$$

e. Suppose you wanted to use Mehra and Prescott's method to estimate the degree of relative risk aversion that would be needed to explain the equity premium in this model economy. What kind of difficulty would you run into then?

a. This is an example of the Bansal-Yaron model, which seeks to explain the equity premium puzzle.
b. The Euler equation in this model depends in general on the trend growth rate x_t, which is stochastic and whose value changes over time. For the Euler equation to hold, the parameter a must then take on a value to ensure that the coefficient of x_t is zero.
c. This computation repeats the one in Bansal and Yaron.

d. According to the formula, we have

$$-\delta\frac{1-\gamma}{1-\delta}=4,\frac{\delta-\gamma}{1-\delta}=-9,$$

which solves as $\delta=0.5,\gamma=4$.

e. In this model, the unconditional variance of consumption growth is not defined as the conditional variance grows to infinity with time.