# SØK3529 Dynamic Macro/Dynamisk makro

Fall/Høst 2017

Final exam

**Text in English**

**OUESTION 1**

Consider a New Keynesian economy described by the following three equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - (i_t - E_t \pi_{t+1} - r_t^n)$$

$$i_t = \rho + \phi \pi_t + v_t$$

where $\pi_t$ is the rate of inflation, $i_t$ the nominal interest rate, and $v_t$ a white-noise monetary policy shock. $\tilde{y}_t \equiv y_t - y_t^n$ is the output gap, where $y_t^n = a_t$ is output with flexible prices, and $a_t$ is a white-noise productivity shock. $r_t^n = \rho - y_t^n + z_t$ is the natural real interest rate, and $z_t$ a white-noise preference shock. $\beta$ is the subjective discount factor and $\rho = -\ln \beta$ the corresponding subjective discount rate. The three white-noise shocks are uncorrelated with each other.

a. Explain each of the three equations verbally, with special emphasis on the logic behind the first equation, including the effects reflected by the parameter $\kappa$. Use mathematical formulations as needed.
b. By substituting from the third equation, describe the dynamics of this economy by a two-equation model on matrix form.
c. Derive the condition under which this model yields a determinate rate of inflation.
d. Under this condition, derive the effect on inflation, output, and the output gap of (i) a technology shock, (ii) a preference shock, and (iii) a monetary policy shock. Comment briefly on your results.

*Answers:*

a. The first equation is the Phillips curve. It is derived from the price setting decisions of individual firms. The firms are assumed to be monopolistic competitors facing demand equations, derived from household optimization, of the form

$$Y(i)_t = \left(\frac{P(i)_t}{P_t}\right)^{-\epsilon} Y_t.$$

Thus, if firms could decide on their prices every period, they would choose

$$P(i)_t = \frac{1}{1 - 1/\epsilon} \psi_{it} \equiv \mu \psi_{it},$$

where $\psi_{it}$ is nominal marginal cost and $\mu$ the optimal markup. However, for reasons of menu costs, costs of attention and decision making, etc., firms don't find it optimal to update their prices each period. As a result, prices are sticky and are updated in a staggered manner. For simplicity, we assume a Calvo mechanism, which stipulates that a share $1-\theta$ of randomly selected firms get to update their prices each period. Because firms are symmetrical, those who get to update prices will choose the same price $P_t^*$. Because these firms are selected at random, the remaining firms will on average charge the average price of the previous period, $P_{t-1}$. Thus, on log form, the overall price level will then be an approximate weighted average of these two prices, i.e.

$$p_t = (1-\theta)p_t^* + \theta p_{t-1}.$$

The firms that get to update their prices will choose a price so as to maximize the expected utility of current and future profits. As an approximation around the steady state, the optimal price will then be a weighted average of current and future nominal marginal costs, where the weights are products of probabilities and subjective discount factors, plus the desired markup:

$$p_t^* = \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t \psi_{t+k|t}.$$

For the case of constant returns, where $\psi_{t+k|t} = \psi_{t+k}$, forward iteration now shows that the optimal price can be written as a weighted average of the current marginal cost, plus the desired markup, and the expected value of the optimal price to be chosen the following period. After some further manipulation, we then obtain the first equation in the question. The case of decreasing returns is more complicated, but yields the same answer.

The appropriate interpretation of this equation starts with the last term. A positive output gap means that, because of some shock, demand exceeds supply as it would have been if all prices have been the same. If all firms had been able to respond immediately to this demand pressure, all prices would have risen by the same amount, so that relative prices would have been unchanged. However, because some firms can't or won't adjust their prices, the increased demand results in higher production for those firms that don't get to raise their prices. The result is then a simultaneous increase in aggregate output and the rate of inflation. However, because of the stickiness, the increase in inflation is lower than in the flexprice case. The parameter $\kappa$ reflects, in an inverse mannder, the degree of price stickiness, indexed by the parameter $\theta$. Thus, the more stickiness, the flatter the Phillips curve because fewer firms get to raise their prices.

The Phillips curve equation may give the impression that inflation also depends on expectations of future inflation. This connection is indirect, however: Because firms' price decisions are based on their expectations of marginal costs in future periods as well as the present one, it will also depend on their expectations of future output gaps. By forward iteration, this effect is summarized in the first term on the right of the equation.

The second equation is easier to interpret. It is derived from the Euler equation of consumer choice:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - \rho).$$

In the absence of investment, government spending, or foreign trade, we have $c_t = y_t$ in equilibrium. Furthermore, for the flexprice case, the equation implies

$$r_t^n = \rho + \sigma E_t \Delta y_{t+1}^n.$$

Substituting this into the above equation now gives the second equation in the question, provided $\sigma = 1$, which the question obviously has assumed.

The third equation is a monetary policy rule that requires the nominal interest rate to respond positively to inflation. The monetary policy shock indicates deviations from this rule.

b. Substituting from the third equation into the second and using the expression given for $r_t^n$, yields

$$\tilde{y}_t = E_t\tilde{y}_{t+1} + E_t\pi_{t+1} - \phi\pi_t + (z_t - a_t - v_t) \equiv E_t\tilde{y}_{t+1} + E_t\pi_{t+1} - \phi\pi_t + u_t.$$

Substituting from the Phillips-curve equation and solving for $\tilde{y}_t$ then gives the first dynamic equation:

$$\tilde{y}_t = \frac{1}{1+\kappa\phi}E_t\tilde{y}_{t+1} + \frac{1-\phi\beta}{1+\kappa\phi}E_t\pi_{t+1} + \frac{1}{1+\kappa\phi}u_t.$$

After substituting from this equation into the Phillips-curve equation, we obtain the second dynamic equation:

$$\pi_t = \frac{\kappa}{1+\kappa\phi}E_t\tilde{y}_{t+1} + \frac{\beta+\kappa}{1+\kappa\phi}E_t\pi_{t+1} + \frac{\kappa}{1+\kappa\phi}u_t.$$

On matrix form:

$$\begin{bmatrix}\tilde{y}_t\\ \pi_t\end{bmatrix} = \left(\frac{1}{1+\kappa\phi}\right)\begin{bmatrix}1 & 1-\beta\phi\\ \kappa & \kappa+\beta\end{bmatrix}\begin{bmatrix}E_t\tilde{y}_{t+1}\\ E_t\pi_{t+1}\end{bmatrix} + \left(\frac{1}{1+\kappa\phi}\right)\begin{bmatrix}1\\ \kappa\end{bmatrix}u_t \equiv A\begin{bmatrix}E_t\tilde{y}_{t+1}\\ E_t\pi_{t+1}\end{bmatrix} + Bu_t.$$

c. Solving this system forward repeatedly, we obtain, after $T$ iterations:

$$\begin{bmatrix}\tilde{y}_t\\ \pi_t\end{bmatrix} = A^{T-t}\begin{bmatrix}E_t\tilde{y}_T\\ E_t\pi_T\end{bmatrix} + \sum_{s=0}^{T-t-1} A^s B E_t u_{t+s}.$$

This equation determines current inflation if $A^{T-t} \to 0$ for $T \to \infty$ and $\begin{bmatrix} E_t \tilde{y}_T \\ E_t \pi_T \end{bmatrix}$ is bounded. Given boudnedness, the remaining condition is satisfied if and only if the characteristic roots of $A$ lie inside the unit circle.

The characteristic polynomial for $A$ is

$$P(\lambda) = |A - \lambda I| = \left(\frac{1}{1 + \kappa\phi}\right)[(1 + \kappa\phi)\lambda^2 - (1 + \beta + \kappa)\lambda + \beta].$$

If the roots are real, they lie between 0 and 1 provided $P(0) > 0, P'(0) < 0, P(1) > 0$, and $P'(1) > 0$. We have $P(0) = \frac{\beta}{1+\kappa\phi} > 0$ and $P'(0) = -\frac{1+\beta+\kappa}{1+\kappa\phi} < 0$. Furthermore,

$$P(1) = \frac{\kappa}{1 + \kappa\phi}(\phi - 1),$$

which is positive for $\phi > 1$. Also,

$$P'(1) = \frac{1}{1 + \kappa\phi}[1 - \beta + \kappa\phi + \kappa(\phi - 1)],$$

which is positive under the same condition.

[However, complex roots are possible if $\phi$ is large enough because the radicand is

$$R = (1 + \beta + \kappa)^2 - 4(1 + \kappa\phi)\beta$$

However, in that case, we find that the mode of the characteristic roots is

$$|\lambda| = \frac{\sqrt{(1 + \beta + \kappa)^2 - (1 + \beta + \kappa)^2 + 4(1 + \kappa\phi)\beta}}{2(1 + \kappa\phi)} = \sqrt{\frac{\beta}{1 + \kappa\phi}} < 1.$$

Note also that, for the radicand to be negative, we must have

$$\phi > 1 + \frac{(1 - \beta + \kappa)^2}{4\beta\kappa} > 1.]$$

Thus, inflation is determined if and only if $\phi > 1$.

d. Note that the white-noise assumption implies

$$E_t u_{t+s} = 0$$

for $s \geq 1$.

Thus, the forward solution derived above implies

$$\begin{bmatrix}\tilde{y}_t \\ \pi_t\end{bmatrix} = \sum_{s=0}^{\infty} A^s B E_t u_{t+s} = Bu_t \equiv \left(\frac{1}{1+\kappa\phi}\right)\begin{bmatrix}1 \\ \kappa\end{bmatrix}(z_t - a_t - v_t).$$

(i) For a technology shock, $a_t = 0, z_t = v_t = 0$. Thus,

$$\pi_t = -\frac{\kappa}{1+\kappa\phi}a, \tilde{y}_t = -\frac{1}{1+\kappa\phi}a, y_t = \tilde{y}_t + y_t^n = -\frac{1}{1+\kappa\phi}a + a = \frac{\kappa\phi}{1+\kappa\phi}a.$$

Output rises with productivity; but the output gap falls because output rises by less than in the flexprice case. The reason is that price stickiness prevents some firms from lowering their relative prices in response to the improved productivity, which in turn dampens the demand for their products. The drop in the output gap is furthermore translated into a drop in inflation via the Phillips curve.

(ii) For a preference shock, $z_t = z, a_t = v_t = 0$. Because technology is unchanged, then so is the natural output level, so the effect on actual output equals the effect on the output gap:

$$y_t = \tilde{y}_t = \frac{1}{1+\kappa\phi}z, \pi_t = \frac{\kappa}{1+\kappa\phi}z.$$

Because of price stickiness, the drop in aggregate demand raises actual output as well as the output gap, and inflation rises via the Phillips curve. If prices had been flexible, the demand increase would have been matched by a drop in the real interest rate ($r_t^n = \rho + E_t \Delta y_{t+1}^n = \rho + z$ in this case) that would have encouraged more saving, i.e. choked off the demand increase.

(iii) For a monetary policy shock, $v_t = v, a_t = z_t = 0$. Like in the case of a preference shock, the flexprice output level is unaffected, so that actual output moves like the output gap:

$$y_t = \tilde{y}_t = \frac{1}{1+\kappa\phi}v, \pi_t = \frac{\kappa}{1+\kappa\phi}v.$$

Although the results are similar to those in the preference shock, the mechanism is different. The exogenous rise in the nominal interest rate also raises the real interest because of price stickiness, and this discourages demand in the current period. Inflation is then affected via the Phillips curve.

**QUESTION 2**

Suppose we have an economy where the productivity in the production of each traded good increases in the production experience of each good (Learning by doing), and also that one can learn from production in other countries. Assume now that the country we are looking at start to receive an exogenous foreign exchange income, for example oil revenues. Discuss how increased incomes from oil affect such an economy, and also how the economy is affected when the incomes from oil is no longer present.

*Answer*

The relevant model to use to answer this question is the model about the Dutch disease in the article by Krugman (1987) on the reading list. The students should first set up the production functions and the learning equations, and describe these. Then they should show how, for a given relative wage, some goods are produced at home and others abroad, and then show how the balanced trade condition gives another relation between relative wages and the allocation of production between home and abroad. Then they should show how the balanced trade condition is affected by increased oil income, and explain the resulting dynamics. Following this, they can conclude how oil affects the relative wages and the allocation of production between home and abroad. Finally, when they answer the question about what happens when incomes from oil is no longer present, they should note that the balanced trade condition shifts back to its original position, but that the economy may not, and that the relative wages and the share of goods produced at home may both decrease.