Exercise 1 (40 points)

In their paper ”A nation of immigrants: Assimilation and economic outcomes in the age of mass migration” published in the Journal of Political Economy in 2014, Abramitzky, Boustan, and Eriksson study the assimilation of European immigrants in the United States labor market during 1850-1913. In this period almost thirty million, mostly European, immigrants moved to the US. The common view among scholars has been that European immigrants held substantially lower-paid occupations than natives upon first arrival, but that they converged with the native born after spending some time in the US. The paper provides new light to this issue.

The authors construct two datasets. They start by obtaining three random samples of migrants and natives from the United States census of the population, one for year 1900, one for the year 1910 and finally one for the year 1920. For the second dataset, they build a panel dataset in the following way. They start from the random sample of natives and migrants they have for 1900 and then they search these individuals by their name, surname and age in the 1910 Census and, again, in the 1920 Census. In doing this exercise, their matching rate is about 25%. In other words the authors are able to follow 25% of the original 1900 sample over time.

Next, they estimate different versions of the following model:

$$\begin{aligned} Earnings_{it} = {} & \beta_0 + \beta_1 Y0\_5_{it} + \beta_2 Y6\_10_{it} + \beta_3 Y11\_20_{it} + \beta_4 Y21\_30_{it} + \beta_5 Y30_{it} + \\ & + \beta_6 age_{it} + \beta_7 age^2_{it} + \beta_8 age^3_{it} + \beta_8 age^4_{it} + \beta_9 after1890 + \theta_t + \alpha_j + a_i + u_{it} \end{aligned} \tag{1}$$

In equation (1), i indicates individuals and t time (1900, 1910, 1920). $Earnings_{it}$ measures the earnings of individuals in 2010 dollars; $Y0\_5$ is an indicator that equals one if the migrant has spent in the US zero to 5 years and zero otherwise, $Y6\_10$ is an indicator that equals one if the migrant has spent in the US 6 to 10 years and zero otherwise, and so on, until $Y30$, which is an indicator that equals one if the migrant has spent in the US more than 30 years and zero otherwise. The omitted category here is being a US born. age_{it} indicates the age of the individual, which appears as a fourth-degree polynomial; $after1890$ is an indicator that equals one if the migrant arrived after 1890. Finally, θ_t indicates Census year fixed effects, α_j indicates country of birth fixed effects and a_i indicates individual fixed effects.

The results from their analysis are reported in Table 1 on page 2. In column (1) and (2), the authors use the pooled data, and the model is estimated by OLS. In column (3), the authors use the panel data and the model is estimated using a fixed-effect estimator.

(a) (5 points) Interpret the coefficients for the years 0 to 20 reported in column (1). What can you conclude about the earnings profile of the immigrants?

(b) (10 points) Focus only on the results reported in column (1) and (2). Why does the immigrant-native earnings gap in the first five years since arrival ($\hat{\beta}_1$) shrink?

(c) (10 points) A young referee suggests that part of the reasons that explain different assimilation profiles could be permanent differences in earnings between immigrant women and other groups, i.e. discrimination against immigrant women. The referee therefore proposes to include in the model of column (3) an indicator for being a female migrant. Provide a brief comment to this proposal.

(d) (15 points) The abstract of the paper reads along these lines: ”Prior cross-sectional work finds that immigrants initially held lower-paid occupations than natives but converged over time. In newly-assembled panel data, we show that, in fact, the average immigrant did not face a substantial earnings penalty upon first arrival and experienced earnings growth at the same rate as natives. Cross-sectional patterns are driven by biases from declining arrival cohort quality and departures of negatively-selected return migrants.” Discuss whether the conclusion that immigrant did not face substantial earnings penalties upon arrival could be questioned. Next, discuss whether cross-sectional patterns could be explained by other reasons besides cohort quality. Remember to base your statements on the concepts learned in class.

Table 1: Age-earnings profile for natives and foreign-born, 1900-1920

	POLS	POLS	FE
	Earnings	Earnings	Earnings
	(1)	(2)	(3)
0-5 Years in US	-1,255.73	-384.48	139.68
	(143.44)	(187.30	(57.96)
6-10 Years in US	-734.51	-2.89	313.81
	(147.44)	(172.05)	(113.61)
11-20 Years in US	-352.93	173.83	175.55
	(131.27)	(132.02)	(200.49)
21-30 Years in US	-294.87	128.44	-79.49
	(142.10)	(138.93)	(150.33)
30+ Years in US	22.41	155.77	78.07
	(184.65)	(178.49)	(186.55)
Arrive after 1890	-	-739.18	-
	-	(106.99)	-
Age controls	Yes	Yes	Yes
θ_t	Yes	Yes	Yes
α_j	Yes	Yes	-
N	205,458	205,458	65,804

Source: selected and modified results from Table 4, Abramitzky et al. (2014), p. 484.
Earnings are measured in 2010 dollars. See text for explanation of the other variables and of the different samples.

Exercise 2 (40 points)

Romer (1993) proposes theoretical models of inflation that imply that more open countries should have lower inflation rates. His empirical analysis explains average annual inflation rates (since 1973) in terms of the average share of imports in gross domestic product since 1973–which is his measure of openness. While Romer does not specify both equations in a simultaneous system, he has in mind a two-equation system:

$$inf = \beta_{10} + \alpha_1 open + \beta_{11} lpcinc + \beta_{12} oil + u_1 \tag{1}$$

$$open = \beta_{20} + \alpha_2 inf + \beta_{21} lpcinc + \beta_{22} lland + u_2, \tag{2}$$

where inf measures the annual inflation rate, $open$ measures imports as % of GDP, $lpcinc$ is the log of 1980 per capita income measured in U.S. dollars, $lland$ is the log of land area of the country measured in square miles and oil is a dummy variable that takes value of one if the country is an oil producer and zero otherwise.

The do-file of the analysis is reported on page 5 and the relative log-file starts on page 6.

(a) (5 points) Consider first the analysis reported on line 17-21 of the do-file. Using a 5% significance level, test for heteroskedasticity.

(b) (10 points) Discuss under which conditions the estimator reported on line 27 and 28 of the do-file can identify the parameters α_1 and α_2. If needed, base your answer on appropriate tests.

(c) (5 points) A commentator suggests to use $land$ in levels, rather than in logs ($lland$), as an instrument for $open$. Provide at least two strategies based on which to decide which regressor to include.

(d) (10 points) How would you test whether the OLS and IV estimates on the equation for $open$ are statistically different?

(e) (10 points) Explain whether you think the estimator used on line 27 of the do-file for inf is consistent and efficient.

Exercise 3 (20 points)

In their article "Do police reduce crime? Estimates using the allocation of police forces after a terrorist attack." published in the American Economic Review (2004), Di Tella and Schargrodsky wirte: "An important challenge in the crime literature is to isolate causal effects of police on crime. Following a terrorist attack on the main Jewish center in Buenos Aires, Argentina, in July 1994, all Jewish institutions received police protection. Thus, this hideous event induced a geographical allocation of police forces that can be presumed exogenous in a crime regression. Using data on the location of car thefts and police forces before and after the attack, we find a large deterrent effect of observable police on crime".

(a) (10 points) Let $CarTheft_{it}$ indicate the number of car thefts in location i at time t, and $NewPolice_{it}$ the number of policemen allocated to location i at time t. Explain which model you think the authors are using to pin down the causal effect of police on crime.

(b) (10 points) Interpret the coefficient(s) you have mentioned at point (a).

```
**********************************************

* Exam Spring 2019: Question 2 Do-File

**********************************************

clear all
cd "/Users/co/Documents/Teaching/Courses Taught/Econometrics/NTNU/Exam"

log using exam_s19, replace text


use OPENNESS.DTA


***
reg inf  oil lpcinc lland
predict res, res
gen ressq= res*res

reg ressq oil lpcinc lland


***
reg inf oil lpcinc lland
reg open  oil lpcinc lland
ivregress 2sls inf lpcinc oil (open =  lland), robust
ivregress 2sls open lpcinc land (inf =  oil), robust



log close
```

```
  ------------------------------------------------------------------------------------------
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         log:  C:\Users\costanzb\Documents\Teaching\Courses Taught\Econometrics\NTNU\Exam\exam_s19.
    log type:  text
   opened on:   6 May 2019, 14:57:10

 1 .
 2 .
 3 . use OPENNESS.DTA

 4 .
 5 .
 6 . ***
 7 . reg inf  oil lpcinc lland

      Source |       SS           df       MS      Number of obs   =       114
-------------+----------------------------------   F(3, 110)       =      2.04
       Model |  3436.23107         3  1145.41036   Prob > F        =    0.1119
    Residual |  61637.1906       110  560.338097   R-squared       =    0.0528
-------------+----------------------------------   Adj R-squared   =    0.0270
       Total |  65073.4217       113  575.870989   Root MSE        =    23.671

------------------------------------------------------------------------------
         inf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         oil |  -6.690162   9.709693    -0.69   0.492    -25.9325    12.55217
      lpcinc |   .6279128   2.084876     0.30   0.764   -3.503823    4.759648
       lland |   2.549812   1.083075     2.35   0.020    .4034102    4.696213
       _cons |  -15.50321   21.49386    -0.72   0.472     -58.099    27.09257
------------------------------------------------------------------------------

 8 . predict res, res

 9 . gen ressq= res*res

10 .
11 . reg ressq oil lpcinc lland

      Source |       SS           df       MS      Number of obs   =       114
-------------+----------------------------------   F(3, 110)       =      0.71
       Model |  23172800.5         3  7724266.82   Prob > F        =    0.5493
    Residual |  1.2002e+09       110  10910513.4   R-squared       =    0.0189
-------------+----------------------------------   Adj R-squared   =   -0.0078
       Total |  1.2233e+09       113  10825922.7   Root MSE        =    3303.1

------------------------------------------------------------------------------
       ressq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         oil |  -619.0514   1354.887    -0.46   0.649   -3304.119    2066.016
      lpcinc |   115.9505   290.9229     0.40   0.691   -460.5903    692.4913
       lland |   208.7068   151.1319     1.38   0.170   -90.80121    508.2148
       _cons |  -2631.407   2999.245    -0.88   0.382   -8575.206    3312.392
------------------------------------------------------------------------------
```

```
12 .
13 .
14 . ***
15 . reg inf oil lpcinc lland

         Source |       SS           df       MS      Number of obs   =       114
   -------------+----------------------------------   F(3, 110)       =      2.04
          Model |  3436.23107         3  1145.41036   Prob > F        =    0.1119
       Residual |  61637.1906       110  560.338097   R-squared       =    0.0528
   -------------+----------------------------------   Adj R-squared   =    0.0270
          Total |  65073.4217       113  575.870989   Root MSE        =    23.671

   ------------------------------------------------------------------------------
            inf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
   -------------+----------------------------------------------------------------
            oil |  -6.690162   9.709693    -0.69   0.492     -25.9325    12.55217
         lpcinc |   .6279128   2.084876     0.30   0.764    -3.503823    4.759648
          lland |   2.549812   1.083075     2.35   0.020     .4034102    4.696213
          _cons |  -15.50321   21.49386    -0.72   0.472      -58.099    27.09257
   ------------------------------------------------------------------------------

16 . reg open  oil lpcinc lland

         Source |       SS           df       MS      Number of obs   =       114
   -------------+----------------------------------   F(3, 110)       =     29.84
          Model |  28607.1395         3  9535.71318   Prob > F        =    0.0000
       Residual |  35150.8507       110  319.553188   R-squared       =    0.4487
   -------------+----------------------------------   Adj R-squared   =    0.4336
          Total |  63757.9902       113  564.230002   Root MSE        =    17.876

   ------------------------------------------------------------------------------
           open |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
   -------------+----------------------------------------------------------------
            oil |   .3989381   7.332499     0.05   0.957    -14.13235    14.93023
         lpcinc |   .5204514   1.574443     0.33   0.742    -2.599724    3.640627
          lland |  -7.566865   .8179095    -9.25   0.000     -9.18777   -5.945961
          _cons |   117.2567   16.23158     7.22   0.000     85.08954    149.4239
   ------------------------------------------------------------------------------

17 . ivregress 2sls inf lpcinc oil (open =  lland), robust

   Instrumental variables (2SLS) regression          Number of obs   =       114
                                                     Wald chi2(3)    =      6.98
                                                     Prob > chi2     =    0.0725
                                                     R-squared       =    0.0349
                                                     Root MSE        =    23.471

   ------------------------------------------------------------------------------
                |               Robust
            inf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
   -------------+----------------------------------------------------------------
           open |  -.3369707   .1500285    -2.25   0.025    -.6310211   -.0429202
         lpcinc |   .8032896   1.527911     0.53   0.599     -2.19136    3.797939
            oil |  -6.555731   3.708423    -1.77   0.077    -13.82411    .7126437
          _cons |   24.00886   11.28069     2.13   0.033     1.899121    46.11861
   ------------------------------------------------------------------------------
   Instrumented:  open
   Instruments:   lpcinc oil lland
```

```
18 . ivregress 2sls open lpcinc land (inf =  oil), robust

   Instrumental variables (2SLS) regression          Number of obs   =        114
                                                     Wald chi2(3)    =      17.30
                                                     Prob > chi2     =     0.0006
                                                     R-squared       =          .
                                                     Root MSE        =     23.952

   ------------------------------------------------------------------------------
                |               Robust
           open |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
   -------------+----------------------------------------------------------------
            inf |   .2660628   1.500991     0.18   0.859    -2.675825    3.207951
         lpcinc |   4.160504   2.557307     1.63   0.104    -.8517257    9.172735
           land |  -.0000138   9.36e-06    -1.47   0.141    -.0000321    4.57e-06
          _cons |    4.73574   39.00215     0.12   0.903    -71.70707    81.17855
   ------------------------------------------------------------------------------
   Instrumented:  inf
   Instruments:   lpcinc land oil

19 .
20 .
21 .
22 . log close
         name:  <unnamed>
          log:  C:\Users\costanzb\Documents\Teaching\Courses Taught\Econometrics\NTNU\Exam\exam_s19.
     log type:  text
    closed on:   6 May 2019, 14:57:10
   ----------------------------------------------------------------------------------------------------
```

APPENDIX G

Statistical Tables

TABLE G.1 Cumulative Areas under the Standard Normal Distribution

z	0	1	2	3	4	5	6	7	8	9
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776

(*continued*)

TABLE G.1 *(Continued)*

z	0	1	2	3	4	5	6	7	8	9
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Examples: If $Z \sim \text{Normal}(0,1)$, then $P(Z \leq -1.32) = .0934$ and $P(Z \leq 1.84) = .9671$.

Source: This table was generated using the Stata® function normprob.

TABLE G.2 Critical Values of the *t* Distribution

		Significance Level				
1-Tailed:		**.10**	**.05**	**.025**	**.01**	**.005**
2-Tailed:		**.20**	**.10**	**.05**	**.02**	**.01**
Degrees of Freedom	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.499
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	30	1.310	1.697	2.042	2.457	2.750
	40	1.303	1.684	2.021	2.423	2.704
	60	1.296	1.671	2.000	2.390	2.660
	90	1.291	1.662	1.987	2.368	2.632
	120	1.289	1.658	1.980	2.358	2.617
	∞	1.282	1.645	1.960	2.326	2.576

Examples: The 1% critical value for a one-tailed test with 25 *df* is 2.485. The 5% critical value for a two-tailed test with large ($>$ 120) *df* is 1.96.

Source: This table was generated using the Stata® function invttail.

TABLE G.3a 10% Critical Values of the *F* Distribution

		Numerator Degrees of Freedom									
		1	2	3	4	5	6	7	8	9	10
Denominator Degrees of Freedom	10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32
	11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25
	12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19
	13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14
	14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10
	15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06
	16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03
	17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00
	18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98
	19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96
	20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94
	21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92
	22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90
	23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89
	24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88
	25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87
	26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86
	27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85
	28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84
	29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83
	30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82
	40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76
	60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71
	90	2.76	2.36	2.15	2.01	1.91	1.84	1.78	1.74	1.70	1.67
	120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65
	∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60

Example: The 10% critical value for numerator $df = 2$ and denominator $df = 40$ is 2.44.

Source: This table was generated using the Stata® function invFtail.

TABLE G.3b 5% Critical Values of the *F* Distribution

		Numerator Degrees of Freedom									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Denominator Degrees of Freedom	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
	16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
	17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
	18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
	21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
	22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
	25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
	26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
	27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
	28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
	29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
	30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
	60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
	90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
	120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
	∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Example: The 5% critical value for numerator *df* = 4 and large denominator *df* (∞) is 2.37.

Source: This table was generated using the Stata® function invFtail.

TABLE G.3c 1% Critical Values of the *F* Distribution

		Numerator Degrees of Freedom									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Denominator Degrees of Freedom	10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
	11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
	12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
	13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
	14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
	15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
	16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
	17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
	18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
	19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
	20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
	21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
	22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
	23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
	24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
	25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
	26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
	27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
	28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
	29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
	30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
	40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
	60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
	90	6.93	4.85	4.01	3.54	3.23	3.01	2.84	2.72	2.61	2.52
	120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
	∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Example: The 1% critical value for numerator *df* = 3 and denominator *df* = 60 is 4.13.

Source: This table was generated using the Stata® function invFtail.

TABLE G.4 Critical Values of the Chi-Square Distribution

	Significance Level		
Degrees of Freedom	.10	.05	.01
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21
11	17.28	19.68	24.72
12	18.55	21.03	26.22
13	19.81	22.36	27.69
14	21.06	23.68	29.14
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.81
19	27.20	30.14	36.19
20	28.41	31.41	37.57
21	29.62	32.67	38.93
22	30.81	33.92	40.29
23	32.01	35.17	41.64
24	33.20	36.42	42.98
25	34.38	37.65	44.31
26	35.56	38.89	45.64
27	36.74	40.11	46.96
28	37.92	41.34	48.28
29	39.09	42.56	49.59
30	40.26	43.77	50.89

Example: The 5% critical value with *df* = 8 is 15.51.

Source: This table was generated using the Stata® function invchi2tail.