Institutt for samfunnsøkonomi

Eksamensoppgave i FIN3006 / FIN8606
Anvendt tidsserieøkonometri

Faglig kontakt under eksamen: Gunnar Bårdsen

Tlf.: 73 59 19 38

Eksamensdato: 6. desember 2016

Eksamenstid (fra-til): 6 timer (09.00-15.00)

Hjelpemiddelkode/Tillatte hjelpemidler: C /Flg formelsamling: Knut Sydsæter, Arne Strøm og Peter Berck (2006): Matematisk formelsamling for økonomer, 4utg. Gyldendal akademiske. Knut Sydsæter, Arne Strøm, og Peter Berck (2005): Economists' mathematical manual, Berlin. Calculator Casio fx-82ES PLUS, Citizen SR-270x, SR-270X College eller HP 30S.

Annen informasjon:

Målform/språk: Bokmål, nynorsk og engelsk

Antall sider (uten forside): 9

Antall sider vedlegg: 0

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig □ **2-sidig** □

sort/hvit □ **farger** □

skal ha flervalgskjema □

Kontrollert av:

Dato Sign

Merk! Studenter finner sensur i Studentweb. Har du spørsmål om din sensur må du kontakte instituttet ditt. Eksamenskontoret vil ikke kunne svare på slike spørsmål.

Bokmål

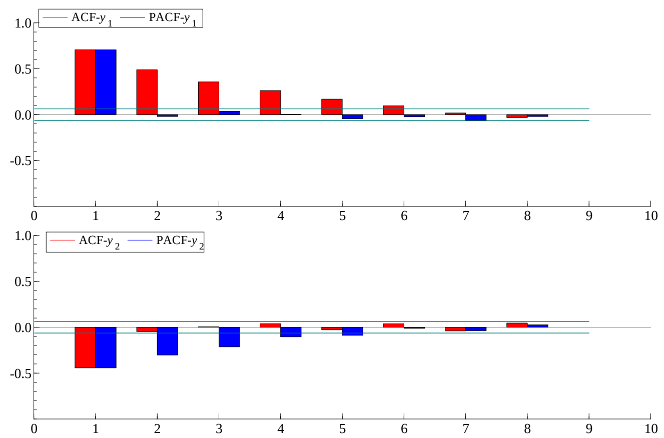
1. Forklar følgende begreper:
   a. "White noise".
   b. "Weak stationarity".
   c. "Skewness".
   d. "Kurtosis".

2. Variabelen y er generert av

$$y_t = a_1 y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2), \qquad y_0 = 0\,,$$

   a. Finn den fullstendige løsningen når $|a_1| < 1$.
   b. Finn forventningen når $|a_1| < 1$.
   c. Finn variansen når $|a_1| < 1$.
   d. Hva er prognosen for y_{t+2} når $|a_1| < 1$?
   e. Finn den komplette løsningen når $a_1 = 1$.
   f. Finn forventningen når $a_1 = 1$.
   g. Finn variansen når $a_1 = 1$.
   h. Hva er prognosen for y_{t+2} når $a_1 = 1$?
   i. Forklar en test av $a_1 = 1$.

3. I denne oppgaven har de to variablene y_1 og y_2 følgende estimerte autokorrelasjoner (ACF) og partielle autokorrelasjoner (PACF):



   Identifiser prosessene for variablene og begrunn svaret.

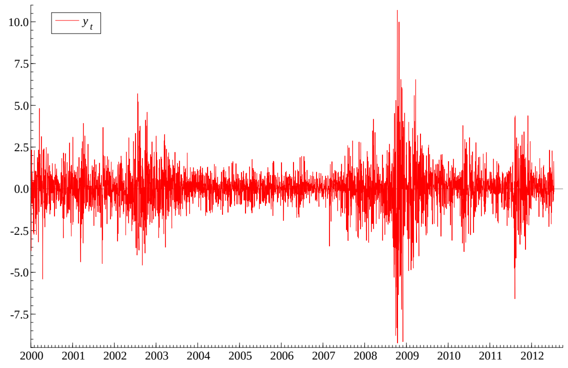
4. I denne oppgaven er de to variablene y_1 og y_2 generert av

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} a_{11,2} & a_{12,2} \\ a_{21,2} & a_{22,2} \end{pmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

   Gå ut fra at begge variablene er stasjonære. Beskriv en test av at y_2 forårsaker y_1.

5. Definér, og forklar forskjellen mellom, Akaike (AIC) og Schwarz (SBIC) sine informasjonskritier.

6. La y_t være avkastningen på en aksjeindeks



a. Du estimerer modellen (standardfeil i paranteser)

$$\text{M1: } \hat{y}_t = \underset{(0.004217)}{-0.0577574} - \underset{(0.01875)}{0.0577574} y_{t-1} - \underset{(0.004217)}{0.0382516} y_{t-2}$$

$$\hat{h}_t = \underset{(0.004035)}{0.0433160} + \underset{(0.006398)}{0.102253} \hat{\varepsilon}^2_{t-1} + \underset{(0.01417)}{0.790003} \hat{h}_{t-1}$$

$$T = 3268, Log - likelihood = -4641.68857, SBIC = 2.8555, AIC = 2.8444.$$

Tolk resultatene ut fra den underliggende teoretiske modellen.

b. Deretter estimerer du modellen

$$\text{M2: } \hat{y}_t = \underset{(0.01380)}{0.0609983} - \underset{(0.01730)}{0.0622358} y_{t-1} - \underset{(0.01771)}{0.0454345} y_{t-2}$$

$$\hat{h}_t = \underset{(0.002959)}{0.00925802} + \underset{(0.01034)}{0.0887103} \hat{\varepsilon}^2_{t-1} + \underset{(0.009600)}{0.909310} \hat{h}_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.7046)}{6.14117}$$

$$T = 3268, Log - likelihood = -4574.36741, SBIC = 2.8168, AIC = 2.8038.$$

Tolk resultatene ut fra den underliggende teoretiske modellen.

c. Deretter estimerer du modellen

$$\text{M3: } \hat{y}_t = \underset{(0.012355)}{0.055331} - \underset{(0.015698)}{0.062170} y_{t-1} - \underset{(0.018136)}{0.045652} y_{t-2}$$

$$\hat{h}_t = \underset{(0.0024999)}{0.008658} + \underset{(0.0095103)}{0.090399} \hat{\varepsilon}^2_{t-1} + \underset{(-)}{0.909601} \hat{h}_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.64750)}{5.994120}$$

$$T = 3268, Log - likelihood = -4576.517, SBIC = 2.8139, AIC = 2.8028.$$

Tolk resultatene ut fra den underliggende teoretiske modellen.

d. Deretter estimerer du modellen

M4: $\hat{y}_t = \underset{(0.012768)}{0.031843} - \underset{(0.015678)}{0.059902} y_{t-1} - \underset{(0.017561)}{0.038175} y_{t-2}$

$$\hat{h}_t = \underset{(0.0024224)}{0.009784} - \underset{(0.0068691)}{0.021886}\,\hat{\varepsilon}^2_{t-1} + \underset{(0.0088351)}{0.933555}\,\hat{h}_{t-1} + \underset{(0.018125)}{0.154179}\hat{\varepsilon}^2_{t-1} I_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.90303)}{7.068088}$$

$$T = 3268, Log - likelihood = -4519.8850, SBIC = 2.7843, AIC = 2.7693.$$

$I_{t-1} = 1$ if $\hat{\varepsilon}_{t-1} < 0$ and 0 otherwise.

Tolk resultatene ut fra den underliggende teoretiske modellen.

e. Deretter estimerer du modellen

M5: $\hat{y}_t = \underset{(0.016401)}{0.035101} - \underset{(0.016072)}{0.060394} y_{t-1} - \underset{(0.019823)}{0.033860} y_{t-2}$

$$\ln \hat{h}_t = -\underset{(0.27496)}{0.328754} - \underset{(0.013894)}{0.128500}\left(\frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}}\right) + \underset{(0.013693)}{0.108887}\left|\frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}}\right| + \underset{(0.0030534)}{0.986043}\ln \hat{h}_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.84578)}{6.846840}$$

$$T = 3268, Log - likelihood = -4522.8942, SBIC = 2.7861, AIC = 2.7712.$$

Tolk resultatene ut fra den underliggende teoretiske modellen.

f. Hvilken modell velger du? Begrunn svaret.

7. Motivér og forklar STAR-modeller.

Nynorsk.

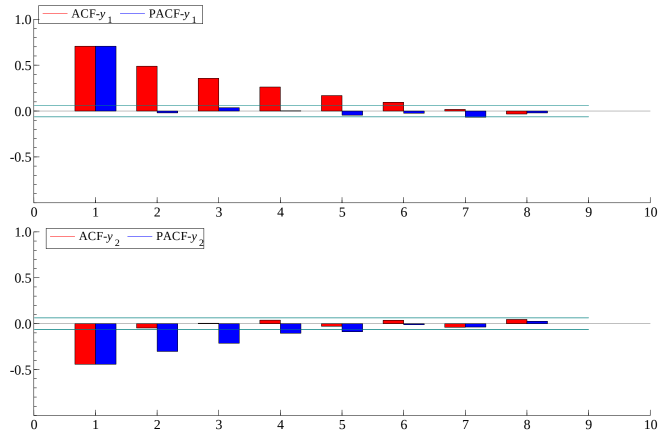
1. Forklar fylgjande omgrep:
   a. "White noise".
   b. "Weak stationarity".
   c. "Skewness".
   d. "Kurtosis".

2. Variabelen y er generert av

$$y_t = a_1 y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2), \qquad y_0 = 0\,,$$

   a. Finn den fullstendige løysinga når $|a_1| < 1$.
   b. Finn forventinga når $|a_1| < 1$.
   c. Finn variansen når $|a_1| < 1$.
   d. Kva er prognosen for y_{t+2} når $|a_1| < 1$?
   e. Finn den fullstendige løysinga når $a_1 = 1$.
   f. Finn forventinga når $a_1 = 1$.
   g. Finn variansen når $a_1 = 1$.
   h. Kva er prognosen for y_{t+2} når $a_1 = 1$?
   i. Forklar ein test av $a_1 = 1$.

3. I denne oppgåva har dei to variablane y_1 og y_2 fylgjande estimerte autokorrelasjonar (ACF) og partielle autokorrelasjonar (PACF):



   Identifiser prosessane for variablane og grunngje svaret.

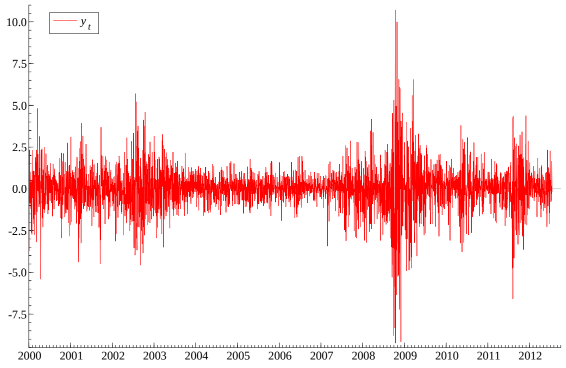
4. I denne oppgåva er dei to variablane y_1 og y_2 genererte av

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} a_{11,2} & a_{12,2} \\ a_{21,2} & a_{22,2} \end{pmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

   Gå ut frå at båe variablar er stasjonære. Beskriv en test av at y_2 forårsakar y_1.

5. Definér, og forklar skilnaden mellom, Akaike (AIC) og Schwarz (SBIC) sine informasjonskritier.

6. La y_t være avkastninga på ein aksjeindeks.



a. Du estimerer modellen (standardfeil i parantesar)

$$\text{M1: } \hat{y}_t = \underset{(0.004217)}{-0.0577574} - \underset{(0.01875)}{0.0577574}y_{t-1} - \underset{(0.004217)}{0.0382516}y_{t-2}$$

$$\hat{h}_t = \underset{(0.004035)}{0.0433160} + \underset{(0.006398)}{0.102253}\hat{\varepsilon}^2_{t-1} + \underset{(0.01417)}{0.790003}\hat{h}_{t-1}$$

$$T = 3268, Log - likelihood = -4641.68857, SBIC = 2.8555, AIC = 2.8444.$$

Tolk resultata ut frå den underliggjande teoretiske modellen.

b. Deretter estimerer du modellen

$$\text{M2: } \hat{y}_t = \underset{(0.01380)}{0.0609983} - \underset{(0.01730)}{0.0622358}y_{t-1} - \underset{(0.01771)}{0.0454345}y_{t-2}$$

$$\hat{h}_t = \underset{(0.002959)}{0.00925802} + \underset{(0.01034)}{0.0887103}\hat{\varepsilon}^2_{t-1} + \underset{(0.009600)}{0.909310}\hat{h}_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.7046)}{6.14117}$$

$$T = 3268, Log - likelihood = -4574.36741, SBIC = 2.8168, AIC = 2.8038.$$

Tolk resultata ut frå den underliggjande teoretiske modellen.

c. Deretter estimerer du modellen

$$\text{M3: } \hat{y}_t = \underset{(0.012355)}{0.055331} - \underset{(0.015698)}{0.062170}y_{t-1} - \underset{(0.018136)}{0.045652}y_{t-2}$$

$$\hat{h}_t = \underset{(0.0024999)}{0.008658} + \underset{(0.0095103)}{0.090399}\hat{\varepsilon}^2_{t-1} + \underset{(-)}{0.909601}\hat{h}_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.64750)}{5.994120}$$

$$T = 3268, Log - likelihood = -4576.517, SBIC = 2.8139, AIC = 2.8028.$$

Tolk resultata ut frå den underliggjande teoretiske modellen.

d. Deretter estimerer du modellen

M4: $\hat{y}_t = \underset{(0.012768)}{0.031843} - \underset{(0.015678)}{0.059902} y_{t-1} - \underset{(0.017561)}{0.038175} y_{t-2}$

$$\hat{h}_t = \underset{(0.0024224)}{0.009784} - \underset{(0.0068691)}{0.021886}\,\hat{\varepsilon}^2_{t-1} + \underset{(0.0088351)}{0.933555}\,\hat{h}_{t-1} + \underset{(0.018125)}{0.154179}\hat{\varepsilon}^2_{t-1} I_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.90303)}{7.068088}$$

$$T = 3268, Log - likelihood = -4519.8850, SBIC = 2.7843, AIC = 2.7693.$$

$I_{t-1} = 1$ if $\hat{\varepsilon}_{t-1} < 0$ and 0 otherwise.

Tolk resultata ut frå den underliggjande teoretiske modellen.

e. Deretter estimerer du modellen

M5: $\hat{y}_t = \underset{(0.016401)}{0.035101} - \underset{(0.016072)}{0.060394} y_{t-1} - \underset{(0.019823)}{0.033860} y_{t-2}$

$$\ln \hat{h}_t = -\underset{(0.27496)}{0.328754} - \underset{(0.013894)}{0.128500} \left(\frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}} \right) + \underset{(0.013693)}{0.108887} \left| \frac{\hat{\varepsilon}_{t-1}}{\sqrt{\hat{h}_{t-1}}} \right| + \underset{(0.0030534)}{0.986043} \ln \hat{h}_{t-1}$$

$$\text{Student t degrees of freedom} = \underset{(0.84578)}{6.846840}$$

$$T = 3268, Log - likelihood = -4522.8942, SBIC = 2.7861, AIC = 2.7712.$$

Tolk resultata ut frå den underliggjande teoretiske modellen.

f. Kva modell vél du? Grunngje svaret.

7. Motivér og forklar STAR-modellar.

English

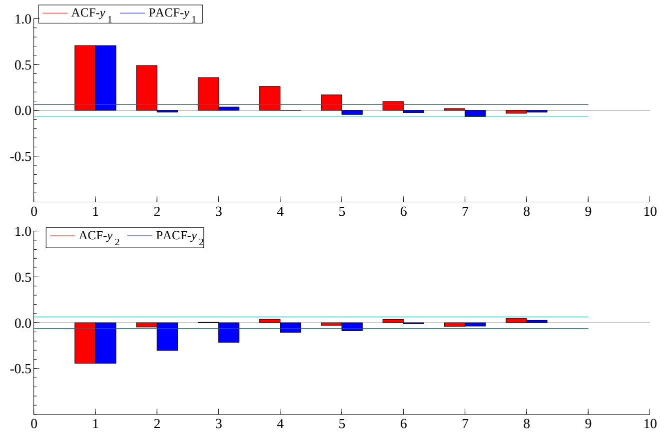
1. Explain the following concepts:
   a. “White noise”.
   b. “Weak stationarity”.
   c. “Skewness”.
   d. “Kurtosis”.

2. The variable y is generated by

$$y_t = a_1 y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2), \qquad y_0 = 0\,,$$

   a. Find the complete solution when $|a_1| < 1$.
   b. Find the expectation when $|a_1| < 1$.
   c. Find the variance when $|a_1| < 1$.
   d. What is the forecast of y_{t+2} when $|a_1| < 1$?
   e. Find the complete solution when $a_1 = 1$.
   f. Find the expectation when $a_1 = 1$.
   g. Find the variance when $a_1 = 1$.
   h. What is the forecast of y_{t+2} når $a_1 = 1$?
   i. Explain a test of $a_1 = 1$.

3. In this question the two variables y_1 og y_2 have the following estimated autocorrelations (ACF) and partial autocorrelations (PACF):



   Identify the processes generating the variables and justify the answer.

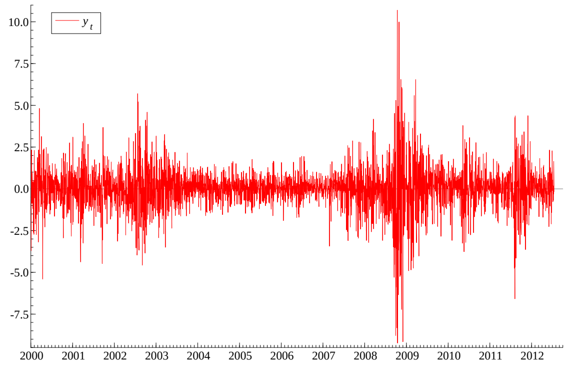
4. In this question the two variable y_1 og y_2 are generated by

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} a_{11,2} & a_{12,2} \\ a_{21,2} & a_{22,2} \end{pmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

   Assume that both variables are stationary. Describe a test of y_2 causing y_1.

5. Define, and explain the difference between, the information criteria of Akaike (AIC) and Schwarz (SBIC).

6. Let y_t be the return of a stock index.



a. You estimate the model (standard errors in parentheses)

$$\text{M1: } \hat{y}_t = \underset{(0.004217)}{-0.0577574} - \underset{(0.01875)}{0.0577574} y_{t-1} - \underset{(0.004217)}{0.0382516} y_{t-2}$$

$$\hat{h}_t = \underset{(0.004035)}{0.0433160} + \underset{(0.006398)}{0.102253} \hat{\varepsilon}_{t-1}^2 + \underset{(0.01417)}{0.790003} \hat{h}_{t-1}$$

$$T = 3268, Log - likelihood = -4641.68857, SBIC = 2.8555, AIC = 2.8444.$$

Interpret the results on the basis of the underlying theoretical model.

b. Then you estimate the model

$$\text{M2: } \hat{y}_t = \underset{(0.01380)}{0.0609983} - \underset{(0.01730)}{0.0622358} y_{t-1} - \underset{(0.01771)}{0.0454345} y_{t-2}$$

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Interpret the results on the basis of the underlying theoretical model.

c. Then you estimate the model

$$\text{M3: } \hat{y}_t = \underset{(0.012355)}{0.055331} - \underset{(0.015698)}{0.062170} y_{t-1} - \underset{(0.018136)}{0.045652} y_{t-2}$$

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d. Then you estimate the model

$$\text{M4: } \hat{y}_t = \underset{(0.012768)}{0.031843} - \underset{(0.015678)}{0.059902} y_{t-1} - \underset{(0.017561)}{0.038175} y_{t-2}$$

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Interpret the results on the basis of the underlying theoretical model.

e. Then you estimate the model

$$\text{M5: } \hat{y}_t = \underset{(0.016401)}{0.035101} - \underset{(0.016072)}{0.060394} y_{t-1} - \underset{(0.019823)}{0.033860} y_{t-2}$$

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$$\text{Student t degrees of freedom} = \underset{(0.84578)}{6.846840}$$

$$T = 3268, Log - likelihood = -4522.8942, SBIC = 2.7861, AIC = 2.7712.$$

Interpret the results on the basis of the underlying theoretical model.

f. What model do you choose? Justify the answer.

7. Motivate and explain STAR-models.