

## Exam in FIN3005 Asset Pricing (Spring 2016)

Make the assumptions you find necessary.

**Problem 1 (75%)** A *Guaranteed Investment Contract* (GIC) is a contract that gives the investor a return on his investment that is equal to the maximum of the return on some underlying asset (typically the return on some stock index) and a minimum guaranteed return. Assume that the time  $T > 0$  value of the underlying asset under the equivalent martingale measure  $Q$  (“the risk neutral probability measure”) is given by

$$S_T = S_0 e^{(r - \frac{1}{2}\sigma^2)T + \sigma z_T},$$

where  $S_0$  is the current value of the underlying asset,  $r$  is the risk-free interest rate,  $\sigma$  is the instantaneous standard deviation of the return on the underlying asset, and  $z$  is a standard Brownian motion. If  $g$  is the guaranteed return per year, the GIC has time  $T$  payoff

$$G_T = \max\left(\frac{S_T}{S_0}, e^{gT}\right).$$

a) Use the martingale approach to find an expression for the current value of the GIC,  $G_0$ .

b) Explain why  $G_0 > 1$ . (*Hint*:  $\frac{S_T}{S_0}$  is a gross return.)

Another GIC gives the investor a minimum guaranteed return *each* year. The time  $T$  value is given by

$$H_T = \prod_{i=1}^T \max\left(\frac{S_i}{S_{i-1}}, e^g\right).$$

c) Find the current value of the GIC,  $H_0$ . (*Hint*:  $\text{cov}\left(\frac{S_t}{S_{t-1}}, \frac{S_{t+1}}{S_t}\right) = 0$ .)

d) Explain why  $H_0 \geq G_0$ .

**Problem 2 (25%)** In class we derived the following expression for the risk-free interest rate when the consumption process follows a stochastic process:

$$r^f = \delta + \gamma\mu - \frac{\gamma}{2}(\gamma + 1)\sigma^2.$$

Give an economic interpretation of the three terms in the expression for the risk-free interest rate.