Examination paper for FIN3005 – Asset Pricing

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Examination date: 16.12.2015
Examination time (from-to): 4 hours (09.00–13.00)
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Permitted examination support material:
- Calculator: Casio fx-82ES PLUS, Citizen SR-270x, SR-270X College or HP 30S.

Language: English
Number of pages (front page excluded): 1
Number of pages enclosed: 0
Exam in FIN3005 Asset Pricing (Fall 2015)

Make the assumptions you find necessary.

Problem 1 (50%) An option matures at time $T > t$. The option has two underlying assets, stock 1 and stock 2. The time $t$ prices of the two stocks are $S_1^t$ and $S_2^t$, respectively. The stocks do not pay dividends. The option is a special type of call option and has an exercise price of $X$. At time $T$ when the option matures, the owner of the option can choose if he wants stock 1 or stock 2 to be the underlying asset.

a) Write down the payoff (value) $\pi_T$ of the option at the time it matures.

b) Use the Martingale approach to find the time $t$ value ($\pi_t$) of the option. (When we in class found an expression for the value of a call option, we found that $Q(S_T > X) = N(d_2)$. You are not expected to find the corresponding $d$-functions here. Stop when you have something similar to $Q(\cdot)$.)

c) Assume now that you would like to estimate the value of the option by using Monte Carlo simulations. To reduce the standard error of the price estimate, you decide to use a control variate. Show how you can estimate the value of the option when you use a call option written on stock 1 as a control variate.

Problem 2 (50%) A representative agent has utility from consumption $u(c) = \ln c$. He will consume at time $t$ and $t+1$. There are two possible states at time $t+1$, $s_1$ and $s_2$. He has a subjective discount factor $\beta$. At time $t$ he has an income $y_0$ and at time $t+1$ he has a state dependent income $y(s)$. Let $y(s_1) = y_1$ and $y(s_2) = y_2$. Let further $pc(s)$ be the state price for state $s$. The two state prices $pc(s_1) = pc_1$ and $pc(s_2) = pc_2$ are known at time $t$. Also the probabilities for state $s_1$ ($\pi_1$) and for state $s_2$ ($\pi_2$) are known. The corresponding discount factors are $m_1$ and $m_2$.

a) Find the optimal consumption $c_0$ at time $t$ and express it in terms of $y_0$, $y_1$, $y_2$, $pc_1$, $pc_2$, and $\beta$.

b) Find the optimal consumption $c_1$ in state $s_1$ and express it in terms of $y_0$, $y_1$, $y_2$, $pc_1$, $pc_2$, $m_1$, and $\beta$.

c) Find the optimal consumption $c_2$ in state $s_2$ and express it in terms of $y_0$, $y_1$, $y_2$, $pc_1$, $pc_2$, $m_2$, and $\beta$. 