NTNU – Trondheim
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Department of Economics

Examination paper for FIN3005 – Asset Pricing

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Permitted examination support material: C / Formelsamling: Knut Sydsæter, Arne Strøm og Peter Berck (2006): Matematisk formelsamling for økonomer, 4utg. Gyldendal akademiske. Knut Sydsæter, Arne Strøm, og Peter Berck (2005): Economists' mathematical manual, Berlin.
Calculator: Casio fx-82ES PLUS, Citizen SR-270x, SR-270X College eller HP 30S.

Language:	English
Number of pages (front page excluded):	3
Number of pages enclosed:	0

Exam in FIN3005 Asset Pricing (Retake, Spring 2015)

Make the assumptions you find necessary.

Problem 1 (25%) Consider a one period model with time points $t = 0, 1$. At time $t = 1$ you will know whether state s_1 og s_2 has occurred. You are given two assets, one risky and one riskfree. Both assets have a price at time $t = 0$ of 1. The time $t = 1$ prices are given by the vectors

$$\mathbf{A}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

and

$$\mathbf{B}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

a) Show *graphically* how you can use the two assets to replicate the payoff of a call option with exercise price $X = 3$ that is written on the risky asset.

b) What is the option price at time $t = 0$?

Assume now that the economy at time $t = 1$ can be in one of three states, s_1, s_2, or s_3. You now also have access to a second risky asset. The time $t = 1$ prices of the three assets are given by the vectors

$$\mathbf{A}_1 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix},$$

and

$$\mathbf{C}_1 = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix},$$

while the time $t = 0$ prices are still 1.

c) In this economy, can you determine the price of the call option from question **a)** and **b)**? If "Yes", calculate the price. If "No", why not.

Problem 2 (25%) On less than two –2– pages, present what you think is important in the 2006 paper "Rare Disasters and Asset Markets in the Twentieth Century" by Robert J. Barro.

Problem 3 (16%) Answer each question with no more than 100 words.

a) Which one of the following statements is inconsistent with the efficient market hypothesis or the rationality of investors? Why?

i. Technical analysis (predicting future prices on the basis of past price patterns) does not persistently beat the market.

ii. In the short run, stock returns have a positive autocorrelation, but in the long run the autocorrelation is negative.

b) Which one of the following statements is false? Why?

i. A mean-variance optimizer with preference $U = R_p - \frac{1}{2}\sigma_p^2$, where R_p and σ_p denote the mean and variance of his portfolio respectively, will never choose the minimum-variance portfolio at optimum.

ii. Three stocks form the efficient frontier in the mean-variance space. Removing one of the assets, which does not lie on the efficient frontier, will not change the efficient frontier.

Problem 4 (34%) Consider a rational agent in a world with two periods, 1 and 2. The agent has initial wealth W in period 1. There are two states in period 2: a "bad" state and a "good" state. The good state occurs with probability s. The agent is a log-utility optimizer and her expected utility is thus $\log c_1 + \beta\left[s \log c_g + (1-s)\log c_b\right]$, where

β – time preference (impatience)
c_1 – the consumption in period 1
c_g – the consumption in the good state of period 2
c_b – the consumption in the bad state of period 2.

Suppose that the agent can invest in a risky asset. For \$1 invested in the risky asset in the first period, the agent will get paid $\$R_g$ in the good state and $\$R_b$ in the bad state of the second period. Assume that $R_g > 1 > R_b$.

a) What is the optimal investment in the asset? What are the optimal consumptions? (You need write down the optimization problem and solve it.)

b) How does the optimal investment change when:

i. β increases?

ii. R_g increases?

ii. s increases?

Suppose that, in addition to the asset above, the agent can also invest in a risk-free asset. For \$1 in the risk-free asset, the agent will get \$$R$ in both states in the second period. Assume that $R_g > R \geq 1 > R_b$. Answer the following questions:

c) Write down the agent's optimization problem (the objective function and budget constraints). Write down the first-order conditions of the optimization problem.

d) Let $s = 0.5$ and $2R = R_b + R_g$. Find the optimal portfolio and the optimal consumptions.

e) In nature, when $s = 0.5$, $2R = R_b + R_g$ implies that the risky asset and risk-free asset have the same expected payoff (or return). Combine this point and give an intuitive explanation of your finding in d).