Solutions FIN3005- spring 2022

Exercise 1.

a) Here,

$$R^f = \frac{1}{E(m)},$$

where

$$m = \beta\frac{u'(c_{t+1})}{u'(c_t)}.$$

Also

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

so that

$$u'(c) = c^{-\gamma}.$$

So,

$$m_u = \beta(\frac{30}{20})^{-2} = 0.96 \cdot (\frac{2}{3})^2 = 0.4267.$$

$$m_d = \beta(\frac{16\frac{2}{3}}{20})^{-2} = 0.96 \cdot (1,2)^2 = 1.3824..$$

$$E(m) = \frac{1}{2}(0.4267 + 1,3824) = 0,9045333.$$

$$R^f = 1.1055$$

$$r^f = 1.1055 - 1 = 0.1055.$$

b)

$$P_i = E[mX]$$

$$P_A = \frac{1}{2}(m_u \cdot X_A^u + m_d \cdot X_A^d) = 5.5893.$$

$$P_B = \frac{1}{2}(m_u \cdot X_B^u + m_d \cdot X_B^d) = 7.9787.$$

$$P_C = \frac{1}{2}(m_u \cdot X_C^u + m_d \cdot X_C^d) = 9,0453.$$

c)

$$E[X_i] = \frac{1}{2}[X_i^u + X_i^d]$$

$$E[X_A] = E[X_B] = 7.5.$$

$$E[X_C] = 10.$$

$$\text{PV}_i = \frac{E[X_i]}{R^f}$$

$$\text{PV}_A = \text{PV}_B = 6.784,$$

$$\text{PV}_C = 9.045.$$

d)

$$r_i = P_i - \mathrm{PV}_i.$$
$$r_A = -1.1947.$$
$$r_B = 1.1947.$$
$$r_C = 0.$$

e)

$$U'(C_t) = (C_t)^{-\gamma} = 20^{-2} = 0.0025.$$
$$E[U'(C_{t+1}] = \frac{1}{2}[U'(C_u) + U'(C_d)] = \frac{1}{2}[30^{-2} + (16\frac{2}{3})^{-2}] = 0.002355556.$$
$$E[U'(C_{t+1})X^i_{t+i}] = \frac{1}{2}[U'(C_u)X^u_i + U'(C_d)X^d_i].$$
$$E[U'(C_{t+1})X^A_{t+i}] = 0.014555556.$$
$$E[U'(C_{t+1})X^B_{t+i}] = 0,020777778.$$
$$E[U'(C_{t+1})X^C_{t+i}] = 0,023555556.$$
$$\mathrm{cov}(U'(X), X^i_{t+1}) = E[U'(C_{t+1})X^i_{t+i}) - E[U'(C_{t+1})]E[X^i_{t+i}]$$
$$\mathrm{cov}(U'(X), X^A_{t+1}) = -0,003111111$$
$$\mathrm{cov}(U'(X), X^B_{t+1}) = 0,003111111$$
$$\mathrm{cov}(U'(X), X^C_{t+1}) = 0.$$
$$\frac{\mathrm{cov}(\beta U'(X), X^A_{t+1})}{U'(C_t)} = -1.1947.$$
$$\frac{\mathrm{cov}(\beta U'(X), X^B_{t+1})}{U'(C_t)} = 1.1947.$$
$$\frac{\mathrm{cov}(\beta U'(X), X^C_{t+1})}{U'(C_t)} = 0.$$

Same as d).

f)

$$rp_i = E[R_i] - R^f$$
$$E(R_A] = \frac{1}{2}(\frac{10}{5.5893} + \frac{5}{5.5893}) = 1.3418.$$
$$E(R_B] = \frac{1}{2}(\frac{5}{7.9787} + \frac{10}{7.9787}) = 0.94.$$
$$E(R_C] = \frac{1}{2}(\frac{10}{9.0453} + \frac{10}{9.0453}) = 1.1055.$$
$$rp_A = 0.2326.$$
$$rp_B = -0.1655.$$
$$rp_c = 0.$$

g)

$$E[mR^i] = 1 \qquad \text{(Basic identity).}$$

$$\mathrm{cov}(m, R^i) = 1 - E(m)E(R^i)$$
$$-R^f \mathrm{cov}(m, R^i) = -R^f + E(R^i)$$
$$E(R^A] - R^f = 0.2326.$$
$$E(R^B] - R^f = -0.1655.$$
$$E(R^C] - R^f = 0.$$

Same as f).

Exercise 2.

	OBX	DNB	Storebrand	Frontline
daily var	0,00013513	0,0003964	0,00066374	0,001298824
daily vol	0,0116245	0,01990987	0,02576311	0,036039206
vol	0,184	0,315	0,407	0,570
cov	0,00013513	6,82E-05	0,00010292	0,000122169
beta	1	0,50	0,76	0,90
old beta		0,8	1,46	1,47

FIGURE 1. Three examples of estimated betas. Assuming 250 trading days per year in row 3. Covariance is calculated from daily data. Old beta is from Gjølberg/Johnsen.

Exercise 3.

Utility is given by

$$U(x) = \frac{1 - e^{-ax}}{a},$$

where $a = 2$. a) Define "total utility" as

$$u = U(x) + \beta EU(\tilde{X})$$

Given the binomial state structure, the state probabilities, and the state dependent endowments, total utility can be calculated as

$$u = U(10) + (\frac{9}{10}U(10) + U(\frac{1}{10}5)) = 0.9999977280.$$

We calculate π so that reduced time 0 consumption together with no uncertainty about time 1 consumption yield the same total utility as the initial situation. I.e.,

$$U(x - \pi) + U(x) = u.$$

This equation has to be solved numerically for π.The solution is $\pi = 3,8489$.

b) No. (known property of $U(x)$),

1. **Exercise 3**

An answer should elaborate on the following:

Campbell and Cochrane introduce habits in the standard model in a clever way. The central ingredient is slow-moving habit, added to the basic power utility function. Habits are external, and depend on history of aggregate consumption, rather than on individual consumption. Habits move slowly in response to consumption (e.g. not proportional to last month's consumption). Habits adapt nonlinearly to the history of consumption. This property always keeps habits below consumption (and avoids undesirable consequences found in other models).

This results in a slowly time-varying, countercyclical risk premium. As consumption declines towards the habit, the curvature of the utility function rises, risky asset prices fall, and expected returns increase.

The model is by construction formulated in a manner that makes the risk free rate constant. This is in correspondence with data and there is no risk free interest rate puzzle.

2. **Exercise 4**

a) Empirical phenomenon. Risk adjusted stock markets returns, e.g. measured by the Sharpe ratio are higher than reasonable risk aversion assumptions and consumption data should imply. The puzzle was established based on US post WW II data, but our textbook claims it holds for most countries. From economic theory one can derive the mathematical relationship

$$\frac{E[R^m] - R^F}{\sigma(R^m)} \leq \gamma\sigma(\Delta),$$

where the left hand side is the Sharpe-ratio of the market return, γ the relative risk aversion coefficient, and $\sigma(\Delta)$, the standard deviation of consumption. From US data (different sources report slightly different numbers, the above numbers are from our textbook) the risk premium $E[R^m] - R^F$ is 8%, market volatility $\sigma(R^m) = 16\%$, so that the Sharpe-ratio is 0.5. The standard deviation of consumption is 1% (again, different sources report different numbers, but they are not very different). These numbers imply that the coefficient should be 50. It is believed that reasonable values of γ are lower than 10, possibly around 2.

It is a puzzle because either something is wrong with the theory or the data. High values of γ (≥ 50) lead to an interest rate puzzle; the risk free interest rate becomes extremely sensitive for small consumption shocks. Possible sources of errors: Extremely high return in all markets last 70 years (just a lucky outcome). Hard to measure true standard deviation of consumption, the standard deviation is typically based on annual data which therefore are smoothed over the year. The model is based on power utility, this function may be too simple to model preferences.

b) Bansal and Yaran (2004) is pretty hard to read. An answer should elaborate on the following: The authors use recursive utility, also called Epstein and Zin (1989) preferences, which allow for a separation between the intertemporal elasticity of substitution (IES) and risk aversion. In addition, they model consumption and dividend growth rates as containing (1) a smal run predictable component, and (2) fluctuating economic uncertainty (consumption volatility). The last point is not very different from the model by Campbell and Cochrane. To allow for time-varying risk premia, they incorporate changes in the conditional volatility of future growth rates. Fluctuating economic uncertainty (conditional volatility of consumption) directly affects price-dividend ratios, and a rise in economic uncertainty leads to a fall in asset prices.

c) The standard condition

$$E_t[m_{t+1}R^i_{t+1}] = 1,$$

where

$$m_{t+1} = \delta(\frac{C_{t+1}}{C_t})^{-\gamma},$$

takes the form

$$E_t[\hat{m}_{t+1}R^i_{t+1}] = 1,$$

where

$$\hat{m}_{t+1} = \delta^\theta G_{t+1}^{\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)},$$

where G_{t+1} is the aggregate gross growth rate of consumption, $R_{a,t+1}$ is the gross return on an asset that delivers aggregate consumption as dividends,

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}},,$$

ψ is the intertemporal elasticity of substitution (IES) parameter (NEW), γ risk aversion parameter (as standard), δ gross, subjective discount rate (as standard).