Exam in FIN3009 Asset Pricing and Portfolio Management (Fall 2023)

Make any assumptions you deem necessary. The weighting of the problems is only indicative.

Problem 1 (30%)

Consider three portfolios, P, Q, and M. The portfolio M is a well-diversified index fund and serves as a proxy for the market portfolio. The table below shows estimates of different characteristics of the portfolio returns. The average excess return is given by $\bar{r}$, the standard deviation of the portfolio returns by σ, the CAPM-beta by β, the alpha from the market model by α, and the standard deviation of the error terms in the market-model regression by σ_e.

	P	Q	M
$\bar{r}$	0.06	0.117	0.08
σ	0.15	0.26	0.16
β	0.80	1.17	1.00
α	0.01	0.027	0.00
σ_e	0.05	0.18	0.00

A university with no previous endowment has received a large donation from one of its former finance students.

a) Which of the portfolios P, Q, or M would you recommend the university to invest in?

The board of the university has decided that the money from the donation will be invested in 15 different portfolios. However, only one of the portfolios P and Q will be chosen.

b) Which portfolio—P or Q—would you recommend to the university board?

Assume now that the university already has an endowment which is invested in a well-diversified portfolio. The donation will either be invested in portfolio P or Q.

c) Which portfolio would you recommend the university to invest in?

d) Go back to your answer to question a). Use the M^2 measure to illustrate why portfolio P is an inferior portfolio choice.

e) If portfolio P were to be the preferred portfolio in question a), what must the correlation ρ have been between the returns on portfolio P and the market portfolio M? What would this correlation imply for the portfolio β?

Problem 2 (35%)

Consider a two period binomial model for the value of a well diversified index fund. In each period, the value of the fund will either increase by 100% or decrease by 50%. There is a 52% chance of an increase in the index fund in each period. The risk-free interest rate is 5.674% per period. The investor has utility function

$$u(W_2) = \frac{1}{1-\gamma} W_2^{1-\gamma},$$

where W_2 is the investor's wealth at the end of the second period and γ is a constant.

a) Show that the investor has a constant coeffisient of relative risk aversion.

Assume $\gamma = 0.9$. This parameter implies that it is optimal for the investor to allocate 50% of the wealth to the index fund and 50% to the risk-free asset.

b) Calculate the expected utility and the certainty equivalent wealth if the investor invests all his funds in the risk-free asset.

c) Calculate the expected utility and the certainty equivalent wealth if the investor invests all his funds in the index fund.

d) Assuming the investor starts out with the optimal allocation at time 0, calculate his expected utility from a buy-and-hold strategy. What is the certainty equivalent wealth?

e) Assuming the investor starts out with the optimal allocation at time 0 and optimally rebalances his portfolio after the first period, what is his expected utility? What is the certainty equivalent wealth?

f) Show that the optimally rebalanced portfolio can be interpreted as being 'short volatility'.

Problem 3 (20%)

An exchange option matures at time T. It gives the owner the option to exchange one asset for another. The payoff at time T is given by

$$\pi_T = \max(S_T^1 - S_T^2, 0),$$

where S_T^1 is the stock price for company 1 and S_T^2 is the stock price for company 2. Under the equivalent martingale measure Q, the stock-price processes are given by

$$dS_t^i = rS_t^i dt + \sigma_i S_t^i dW_t^i, \qquad i = 1, 2,$$

where r is the constant risk-free interest rate, σ_i is a positive constant, and W^i is a standard Brownian motion. Assume that W^1 and W^2 are uncorrelated.

a) Use the martingale apporach to find the time 0 value π_0 for the option (you do not have to calculate the corresponding d_1 and d_2 functions, cf. the Black and Scholes model).

b) Sketch how you would use Monte Carlo simulations to estimate the price of the exchange option.

c) Assume that the returns on stock 1 and 2 are positively correlated and $S_0^1 < S_0^2$. How does this change in correlation affect the price of the option?

Problem 4 (15%)

Let the domestic risk-free interest rate be $r_{t,T} = 0.05$. The foreign risk-free interest rate $r_{t,T}^* = 0.07$. The current exchange rate $S_t = 10$. The forward price in the market is $F_{t,T} = 10.19$.

Is the forward price arbitrage free? If not, show how to exploit any arbitrage opportunity.