

Final Exam
Applied Time Series Econometrics, FIN3008
Fall 2023 NTNU

- Please type or write your answers clearly and show your steps to receive partial credit
- For Question 1 -3, it is useful to use equations in your answer, and make sure to clearly label the equations and models.
- If your answers involve no equations at all in Question 1-3, then you need to explain very well in words to get full credit
- For Question 4-5, there is no need to show your steps.

Question 1 (10 points)

Define (weak) stationarity and explain why it is important in time series analysis. Explain what is a unit root test and how to set up the null and alternative hypothesis for a unit root test.

Question 2 (10 points)

Discuss the difference between trend-stationary and difference-stationary time series. How can differencing be used to achieve stationarity in a time series? Explain what it means for a time series to be integrated of certain order and discuss how we handle integrated series.

Question 3 (10 points)

Briefly describe the purpose and interpretation of forecast error metrics. List and explain at least two measures of the forecast error metrics we discussed in the lecture.

Question 4 (50 points)

Let's consider a stationary autoregressive process of order 1, denoted as AR(1), given by

$$x_t = \phi x_{t-1} + \varepsilon_t$$

where x_t is the value of the time series at time t , ϕ is the autoregressive parameter, and ε_t is a white noise error term at time t with mean 0 and variance σ_ε^2 .

- (1) Find the mean and the variance of the above AR(1) process.
- (2) Derive the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the above AR(1) process.
- (3) Suppose now you don't know whether the above AR(1) model is stationary and please find out the necessary and sufficient conditions for this model to be stationary.
- (4) Now you are using this AR(1) model to make prediction. Derive the one-step forecast, $E_t(x_{t+1})$. Show that as the forecast horizon goes to infinity, the limit of the conditional mean converges to its unconditional mean.
- (5) The process exhibits the feature of mean reversion, and in finance it is often useful to express it as the half-life. Explain what is half-life and derive the half-life of AR(1).

Question 5 (10 points)

ARMA properties: which of the following statements are TRUE?

- (A) An MA(q) can be express as an AR(∞) if it is invertible
- (B) A random walk series will have zero PACF except at lag 1
- (C) The (unconditional) mean of an ARMA process (assume staionarity) will depend only on the intercept and on the AR coefficients and not on the MA coefficients
- (D) An AR(p) can be written as an MA(∞) if it is stationary

Question 6 (10 points)

Which of the following sets of characteristics would usually best describe an autoregressive process or order 3, that is an AR(3)?

- (A) A slowly decaying PACF and an ACF with 3 significant spikes
- (B) An ACF and a PACF with 3 significant spikes
- (C) A slowly decaying ACF and PACF
- (D) A slowly decaying ACF and a PACF with 3 significant spikes