Exam FIN 3005 Asset pricing – December 15, 2022

Exercise 1 — 15 %

Short and concise answers are rewarded - maximum one page.

a) What is the equity premium puzzle?

b) Why is it a puzzle?

c) Are there any resolutions to the puzzle?

Exercise 2 — 40 %

Assume that an agent has power utility of consumption. That is, the utility of consumption C is given by

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma},$$

where γ is a non-negative constant.

a) Calculate

$$\text{RRA} = -\frac{U''(C)}{U'(C)}C.$$

What is the interpretation of RRA?

Assume now that there are two dates, time t and time $t+1$. Total utility for consumption at both dates is given by

$$U(C_t) + e^{-\delta}\mathbb{E}[U(C_{t+1})],$$

where $\delta > 0$ is the subjective discount rate, $U(\cdot)$ is the given power utility function, and $\mathbb{E}[\cdot]$ is the usual expectation operator.

Assume that consumption growth is given by

$$\frac{C_{t+1}}{C_t} = e^{\tilde{\Delta}},$$

where $\tilde{\Delta} \sim \mathcal{N}(\mu.\sigma^2)$. The stochastic discount factor is defined as

$$m_{t+1} = e^{-\delta}\frac{U'(C_{t+1})}{U'(C_t)}.$$

b) Calculate m_{t+1} for the given consumption growth and utility function.

c) What is the probability distribution of m_{t+1}?

d) Derive an expression for the (net) risk free interest rate $r^f = \ln(R^f)$.

Hint 1: $\mathbb{E}[m_{t+1}] = \frac{1}{R^f}$, where R^f is the gross risk free interest rate.
Hint 2: For $\tilde{X} \sim \mathcal{N}(a.b^2)$, $\mathbb{E}[e^{\tilde{X}}] = e^{a+\frac{1}{2}b^2}$.

Assume that $\mu = \sigma = 0.01$, $\gamma = 2$, and $\delta = 0.02$.

e) Calculate the value of r^f.

Due to increased macroeconomic uncertainty assume that the expected consumption growth changes to $\mu = 0$, and that the volatility of consumption changes to $\sigma = 0.02$. Also, γ changes to $\gamma = 4$.

f) What is the value of r^f under the altered assumptions?

Exercise 3 — 45%

Use a one period binomial model with two future states at time 1, *up* and *down*. The market index, which at time 0 has value $S_0 = 500$, takes the values $S_u = 624$ and $S_d = 468$ in the states *up* and *down*, respectively. Assume that the gross risk free interest rate is $R^f = 1.04$.

a) Check whether the state prices, π_u and π_d for states up and down, respectively, in this model are

$$\pi_u = \frac{25}{78}, \qquad \pi_u = \frac{25}{39}.$$

A put option on the market index with exercise price K has time 1 payoff $\max(K - S_1, 0)$, where $S_1 = S_u$ in state up, and $S_1 = S_d$ in state down. Assume that $K = 507$.

b) Calculate P_0, the time 0 value of the put option from the state prices.

Let m_u and m_d be the value of the stochastic discount factor in the states up and down, respectively.

c) Explain why

$$\pi_u = pm_u, \qquad \text{and} \qquad \pi_d = (1-p)m_d,$$

where p is the probability for state up.

Assume that $p = \frac{5}{13}$.

d) Calculate the expected gross return $\mathbb{E}[R_m]$ of the market index.

e) Calculate m_u and m_d.

f) Calculate the time 0 price of the put option from the formula

$$P_0 = \mathbb{E}[mX],$$

where $m = m_u$ in state up, and $m = m_d$ in state down. Also, $X = \max(K - S_1, 0)$.

g) Your boss wants you to also calculate the price of the option by using the capital asset pricing model (CAPM). Would you expect to get the same option price by using the CAPM as you got in question b) ? Why/why not?

Denote the time 0 price of the put option calculated by CAPM by π. *In the following four questions the answers must be stated in terms of* π.

h) Calculate the gross return R_i of the put option as a function of the unknown time 0 price π.

i) Calculate $\text{Cov}(R_m, R_i)$, the covariance between R_m and R_i, and $\text{Var}(R_m)$, the variance of the return of the market index.

Hint 1: $\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$.
Hint 2: $\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

j) Calculate the β of the put option.

Hint:

$$\beta = \frac{\text{Cov}(R_m, R_i)}{\text{Var}(R_m)}.$$

k) Calculate $E[R_i]$, the expected return of the option from CAPM.

Hint: CAPM states that

$$E[R_i] = R^f + \beta[\mathbb{E}[R_m] - R^f].$$

By definition $E[X] = \pi E[R_i]$, where $X = \max(K - S_1, 0)$ is the (random) payoff of the put option.

l) Calculate π by equating $E[R_i] = \frac{E[X]}{\pi}$ from the above equation with the expression for $E[R_i]$ from CAPM in k). Comment whether $\pi = P_0$.