Play: propagation direction $+z$

Wavelength from $k = \frac{2\pi}{\lambda} = 1.26 \cdot 10^5$ → $\lambda = 499$ nm

Frequency from $\omega = 2\pi f = 3.77 \cdot 10^7$ → $f = 6.00 \cdot 10^7$ Hz

b) $\vec{E}(x,y,0,0)$ we know $\vec{k}, \vec{E}, \vec{B}$ form right-handed system and

$$|\vec{E}| \Rightarrow \text{c} \Rightarrow \text{speed of light} \Rightarrow \downarrow \downarrow$$

$$\Rightarrow \vec{B}(x,y,z,t) = \frac{1}{c^{\gamma}} \left( 3 \hat{x} + \hat{y} \right) \text{co}(kz - \omega t)$$

c) The Jones' vector is defined from $\vec{J} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$ since $E_x$ & $E_y$ in phase

$$\vec{J} = \frac{1}{\lambda_{\text{no}}} \begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{linearly polar. as shown in (16)}$$

$$\Delta \phi = \phi_\gamma - \phi_\delta$$

$$e^{\frac{i\pi}{4}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -3 \end{bmatrix} = e^{\frac{i\pi}{4}} \left( \frac{1}{\sqrt{10}} \begin{bmatrix} 1 \\ -3e^{\frac{i\pi}{2}} \end{bmatrix} = e^{\frac{i\pi}{4}} \right) \frac{1}{\sqrt{10}} \begin{bmatrix} 3e^{\frac{i\pi}{2}} \end{bmatrix}$$

d) $\text{QWP 5} \%$ rotation

$$\text{INPUT} \quad \text{LEFT-HANDED KINE} \quad +90^\circ$$

$\text{Elliptically Polarized Left-handed}$

$E_y: \frac{\sqrt{2}}{100}$

$E_x: \frac{\sqrt{2}}{100}$

$\phi = \frac{\pi}{2}$ & $\Delta \phi \neq 0, \pi$
Case I \[ L_1 \]

**Formula:**

\[
\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f_1}
\]

\[
\frac{1}{s_0} + \frac{7}{15} = \frac{1}{f_1} \Rightarrow \frac{1}{s_0} = \frac{2}{15} = \frac{1}{10} \Rightarrow f_1 = 10 \text{ cm}
\]

\[ n_1 = n_T = -\frac{s_i}{s_o} = -\frac{80}{15} = -\frac{16}{3} \Rightarrow n_1 = 4 \text{ or } 1 \]

**Case II** \[ L_1(f_{10}) \quad L_2(20) \]

**Diagram:**

1. **Thin Lens from IMAGE TWO**
   
   \[
   \frac{1}{s_1} + \frac{1}{20} = \frac{1}{f_2} \Rightarrow \frac{1}{20} = \frac{1}{f_2} \Rightarrow f_2 = -20 \text{ cm}
   \]

2. **Newton's LENS**

   \[ M_2 = -\frac{s_i}{s_o} = -\frac{20}{-10} = 2 \]

3. **\[ M_{14} = n_1 \cdot n_2 = -2 \cdot 2 = -4 \text{ or } \]

   \[ L_2 \quad f_2 = -20 \text{ cm} \]

\[ \Rightarrow n_{T1} - n_{T2} \]
Using Ray Transfer Matrix

Two Water Remaining \( d \) \( \checkmark = \checkmark = \checkmark \)

\[
\begin{pmatrix}
1 - \frac{d}{f_2} & -\frac{1}{f_1} & -\frac{1}{f_2} + \frac{d}{f_1 f_2} \\
1 - \frac{d}{f_1} & 1 - \frac{d}{f_1}
\end{pmatrix}
= \begin{cases}
d = 20 \\
f_1 = 10 \\
f_2 = -20
\end{cases}
\]

\[
\begin{pmatrix}
1 - \frac{20}{20} & -\frac{1}{10} - \frac{1}{20} + \frac{20}{10(-20)} \\
20 & 1 - \frac{20}{10}
\end{pmatrix}
= \begin{pmatrix}
20 & -\frac{7}{20} \\
20 & -1
\end{pmatrix}
\]

Objective

\[
\begin{pmatrix}
1 & 0 \\
\frac{5}{4} & -1
\end{pmatrix}
= \begin{pmatrix}
\frac{-1}{4} & \frac{-7}{20} \\
\frac{-5}{4} + 5 & \frac{-15}{20} - 1
\end{pmatrix}
\]

\[
N_T = \frac{-1 - 20}{20} - 1 = -4
\]

\[
\frac{-5}{4} + 5 = 0 \Rightarrow \frac{5}{4} = 20
\]
3)

\[ \theta_1 \]

\[ \text{air} \ n = 1 \quad n_2 \quad n_1 = 1.7 \quad n_3 = 1.6 \]

\[ \sin \theta_2 = \frac{n_2}{n_1} \]

\[ \sin \theta_1 = \theta_2 \quad \theta_1 \]

\[ \theta_1 \]

**Geometry and Relationship:**

\[ n_1 \sin (\theta_1 - \theta_2) = n_2 \sin 90^\circ \]

\[ \sin \theta_2 = n_1 \cos \theta_2 = n_2 \implies \sin \theta_2 = \frac{n_2}{n_1} \]

**Incident Refl:**

\[ 1 - \sin \theta_1 = n_1 \sin \theta_2 = n_1 \sqrt{1 - \sin^2 \theta_2} \]

\[ = n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2} \]

\[ \max \theta_1 \text{ is } \theta = \arcsin \left( \sqrt{n_1^2 - n_2^2} \right) \]

\[ \text{with } n_1 = 1.7, n_2 = 1.5 \implies \theta_{\max} = 51.1^\circ \]

b) Choose polarization in the plane of incidence, then choose incident angle and Brewster angle, i.e.,

\[ 1 - \sin \theta_1 = 1.7 \sin \theta_2 \text{ with } \theta_1 + \theta_2 = 90^\circ \]

\[ \tan \theta_1 = 1.7 \implies \theta_1 = 59.5^\circ \]
Recall reflection as function of incident angle

\[ \Theta_1 (\text{deg}) \]

\[ \Theta_{\text{max}} = 51.1^\circ \]

Choose polarization \( \parallel \) (in plane of incidence)

@ 53.1° to be as close to \( \Theta_0 \) as possible

\[ n_1 \sin \Theta_1 = n_2 \sin \Theta_2 \]

\[ \sin 53.1^\circ = 1.17 \times \sin \Theta_2 \]

\[ \frac{53.1^\circ}{\sin \Theta_2} = 28.1 \]

Check!

\[ \Gamma_{\bot} (\Theta = 53.1^\circ) = \frac{n_1 \cos \Theta_1 - n_2 \cos \Theta_2}{n_1 \cos \Theta_1 + n_2 \cos \Theta_2} \]

\[ = \frac{1.7 \cos 53.1^\circ - 1.17 \cos 28.1^\circ}{1.7 \cos 53.1^\circ + 1.17 \cos 28.1^\circ} = 0.0728 \]

\[ \Gamma_{\|} (\Theta = 53.1^\circ) = \frac{n_2 \cos \Theta_1 - n_1 \cos \Theta_2}{n_2 \cos \Theta_1 + n_1 \cos \Theta_2} \]

\[ = \frac{1.7 \cos 53.1^\circ - 1.17 \cos 28.1^\circ}{1.7 \cos 53.1^\circ + 1.17 \cos 28.1^\circ} = 0.1386 \]
we have a thick lens composed of two surfaces and free propagation two.

**Question:**

a) Find $M_{\text{sys}}$

$$M_{\text{sys}} = \begin{pmatrix} 1-D_2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{d}{n} & 1 \end{pmatrix} \begin{pmatrix} 1 & -D_1 \\ 0 & 1 \end{pmatrix} = D_1 = \frac{n_1-n_0}{R_1}$$

Surface II

$$d = 2$$

Surface I

$$\frac{1.5 - 1.0}{5} = 1 \Rightarrow 10$$

$$D_2 = \frac{n_0 - n_1}{R_2} = 0$$

$$R_2 \to \infty$$

$$M_{\text{sys}} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{d}{n} \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{10} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{10} \\ \frac{d}{n} & \frac{1}{10} + 1 \end{pmatrix}$$

Check!!

$$\text{det } M_{\text{sys}} = \frac{1}{15} - \left(-\frac{1}{10} \cdot \frac{1}{10} \right) = \frac{13}{15} + \frac{1}{20} = \frac{2.17 + 0.05}{20} = 1 \text{ cm}$$

**Focal Lengths:**

$$F_1 : p = \frac{a_{11}}{a_{12}} = -10 \text{ cm left of 1st surface}$$

$$F_2 : q = \frac{a_{11}}{a_{12}} = \frac{13 - 10}{15} = \frac{3}{15} = 0.2 \text{ cm behind 2nd surface}$$

$$H_1, v = \frac{1}{u} = \frac{a_{11}}{a_{12}} = 0 \text{ at 1st surface}$$

$$H_2, w = \frac{s}{u} = \frac{1 - a_{22}}{a_{12}} = \frac{1 - \frac{1}{10}}{\left(-\frac{1}{10}\right)} = \frac{2}{15} \cdot 10 = -\frac{2}{15} = -\frac{4}{3}$$

$$= -1.33 \text{ cm}$$

**Left of 2nd surface**
b) \[
\begin{pmatrix}
1 & -\frac{1}{10} \\
\frac{5}{1} & -\frac{8}{10} + \frac{17}{15}
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
1 - \frac{s_0}{10} & -\frac{1}{10} \\
\frac{s_0}{1} + s_0 \left( -\frac{s_0}{10} + \frac{17}{15} \right) & -\frac{s_0}{10} + \frac{17}{15}
\end{pmatrix}
\]

= 0 \rightarrow \text{imagine condition}
Virtual Image  
\[ s'_v = -25 \text{ cm} \quad (\text{neglect thickness of lens}) \]

\[ H_T = -\left( \frac{-23}{15} \right) + \frac{17}{15} = \frac{101}{15} \approx 7.3 \text{ cm} \]

\[ s_0 = \frac{-25 + \frac{4}{3} + s_0 \left( -\frac{-23}{15} + \frac{17}{15} \right)}{1/3} = 0 \]

\[ s_0 = \frac{2130}{703} \approx 3.0 \text{ cm} \]
$\lambda = 546.1 \text{ nm} \quad \Delta \lambda = 0.050 \text{ nm}$

**Problem 5**

a) $l_c = c \cdot \nu_0$ where $\Delta \nu = \frac{1}{\nu_0} \quad$ Spectral Width

In the Michelson interferometer, find the range where fringes are visible (as the whole range may be invisible). Set the mirror at the middle position. With fringes barely seen, scan the mirror through the whole range and read the distance until fringes disappear again. The optical path $s = 2 \cdot d = \text{coherence length}$.

\[ c = \frac{\lambda}{\nu} \quad \Rightarrow \quad \nu = \frac{c}{\lambda} \quad \Rightarrow \quad \Delta \nu = \frac{1}{\lambda^2} \quad \Rightarrow \quad \Delta \lambda = \frac{c}{\lambda^2} \]

Find spectral width in $\Delta \lambda$

\[ \left( \frac{546.1 \times 10^{-9}}{0.050 \times 10^{-9}} \right)^2 = 5.96 \times 10^{-2} \text{ m} \]

So, expected $s = 2d = 5.96 \text{ mm} \Rightarrow d = 2.98 \text{ cm}$

(This corresponds to $\frac{l_c}{\lambda_0} = 1000$ fringes)
PS-b) For far-field diffraction, the quadratic exponential in (Fresnel approx.) diffraction

\[ u(x, y) = \frac{1}{i2\pi} e^{ikz} \left( \frac{1}{4\pi} \left( \frac{x^2 + y^2}{z^2} \right) \right) \left( \sqrt{\left[ u(0, y) e^{ikx} \right]} \right) e^{i\frac{2\pi}{\lambda z} (y + y')} dy'dy' \]

Only consider one-d (slit) field distribution at aperture plane

\[ \frac{\pi \cdot \lambda^2}{x^2} \ll \pi \]
\[ \text{Hence: } \frac{\lambda}{x} = \text{slit width} + \text{slit separation} \approx 50 \mu m \]
\[ \lambda = 546.1 \times 10^{-9} \text{ m} \quad z = 1 \text{ m} \]
\[ \Rightarrow \frac{\pi \cdot (50 \mu m)^2}{(546.1 \mu m)} \ll 0.019 \ll \pi \text{ by factor } 22.5 \text{ ou il!} \]

Q) We can use the formula for N-slit

\[ I = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin \alpha}{\sin \alpha} \right)^2 \]

\[ \alpha = \frac{1}{2} k \alpha \sin \theta, \text{ a slit separation} \]
\[ \beta = \frac{1}{2} k \beta \sin \theta, \text{ a slit width} \]

\[ \frac{\sin 2\alpha}{\sin \alpha} = \frac{2 \sin \alpha \cos \alpha}{\sin \alpha} = 2 \cos \alpha \]

\[ I = 2I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha \]
i) EXAMINE DIFFRACTION term: \[ \sin \beta \quad \text{(minimum)} \]
\[ \beta = \frac{1}{2} \frac{k a}{b} \sin \theta = \frac{m}{\lambda} \pi \]
\[ \Rightarrow \frac{1}{2} \frac{k a}{b} \sin \theta = \frac{m}{\lambda} \pi \]
\[ \Rightarrow m = \frac{k a}{b} \sin \theta \]

\[
\begin{array}{cccccc}
 m & \sin \theta_{m,0} & \theta_{m,0} \text{(rad)} & \theta_{m,0} \text{(deg)} \\
 0 & 0 & 0 & 0 & 0 \\
 \pm 1 & \pm 0.05461 & \pm 0.05461 & \pm 1.017 & \pm 1.017 \\
 \pm 2 & \pm 0.10922 & \pm 0.10922 & \pm 2.033 & \pm 2.033 \\
 \pm 3 & \pm 0.16383 & \pm 0.16383 & \pm 3.050 & \pm 3.050 \\
 \pm 4 & \pm 0.21844 & \pm 0.21844 & \pm 4.067 & \pm 4.067 \\
 \text{etc} & & & & \\
\end{array}
\]

ii) EXAMINE INTERFERENCE PART
\[ \alpha = \pm m \pi \]
\[ \Rightarrow \text{maxima} \quad m = 0, 1, 2, \ldots \]
\[ \frac{1}{2} k a \sin \theta = \pm m \pi \]
\[ \Rightarrow \sin \theta_{m,a} = \pm m \pi \]

\[
\begin{array}{cccccc}
 m & \sin \theta_{m,a} & \theta_{m,a} \text{(rad)} & \theta_{m,a} \text{(deg)} & \frac{2k}{\alpha} \left( \frac{\sin \beta}{\beta} \right)^2 & \beta \\
 0 & 0 & 0 & 0 & 2k & 0 \\
 \pm 1 & \pm 0.05461 & \pm 0.05461 & \pm 1.017 & 2.033 & 0.05461 \\
 \pm 2 & \pm 0.10922 & \pm 0.10922 & \pm 2.033 & 2.033 & 0.10922 \\
 \pm 3 & \pm 0.16383 & \pm 0.16383 & \pm 3.050 & 2.033 & 0.16383 \\
 \pm 4 & \pm 0.21844 & \pm 0.21844 & \pm 4.067 & 2.033 & 0.21844 \\
 \text{etc} & & & & & \\
\end{array}
\]

\[ \text{MINIMA comes in between the MAXIMA} \]
\[ \frac{1}{2} k a \sin \theta_m = \pm \left( \frac{2m+1}{2} \right) \]
\[ m = 0, 1, 2, \ldots \]