Examination paper for FY2450 Astrophysics

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Phone: 94820834

Examination date: 01-06-2015
Examination time: 09:00 – 13:00
Permitted examination support material: Calculator, translation dictionary, printed or hand-written notes covering a maximum of one side of A5 paper.

Other information: The exam is in three parts. Part 1 is multiple choice. Answer all questions in all three parts. The percentage of marks awarded for each question is shown. An Appendix of useful information is provided at the end of the question sheet.

Language: English
Number of pages: 9 (including cover)
Number of pages enclosed: 0

Checked by:

________________________________________
Date Signature
Part 1. Total 45%

Part 1 is multiple choice. 3 marks will be awarded for each correct answer. No marks will be awarded for an incorrect or missing answer. On your answer sheet draw a table that looks something like this:

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
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<th>4</th>
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<th>13</th>
<th>14</th>
<th>15</th>
</tr>
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<tbody>
<tr>
<td>Answer</td>
<td></td>
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</tbody>
</table>

and insert your answer (A, B, C, D or E) in the appropriate box. Only select one answer. Select the answer closest to your answer with the correct units. Only the answers will be marked. You may use the data in the Appendix to answer these questions if required.

1. What is the approximate right ascension and declination of the Sun on September 23rd (the northern hemisphere autumn equinox)?
   A. 12 hr, +0°
   B. 0 hr, +23.5°
   C. 23.5 hr, +0°
   D. 0 hr, +0°
   E. 12 hr, -23.5°

2. The absolute magnitude and colour of two stars (X and Y) are plotted on a Hertzsprung-Russell diagram. Star X lies directly above Star Y on the diagram. Which of the following is true of the two stars?
   A. Star Y is more luminous than star X.
   B. The photosphere of star Y is hotter than that of star X.
   C. Star Y is smaller than star X.
   D. Star Y is more massive than star X.
   E. All of the above.

3. The absolute magnitude and colour of two stars (X and Y) are plotted on a Hertzsprung-Russell diagram. Star X lies directly to the right of Star Y on the diagram. Which of the following is true of the two stars?
   A. Star Y is more luminous than star X.
   B. The photosphere of star Y is cooler than that of star X.
   C. Star Y is smaller than star X.
   D. Star Y is bigger than star X.
4. Estimate the distance to a star of spectral type A0V if its apparent V-band visual magnitude is measured at +11 and its observed (B-V) colour = 0.

A. 10 pc  
B. 2190 pc  
C. 115 pc  
D. 1150 pc  
E. $1.3 \times 10^5$ pc

5. Estimate the distance to a star of spectral type A0V if its apparent V-band visual magnitude is measured at +11 and its observed (B-V) colour = +0.50.

A. 56 pc  
B. 115 pc  
C. 912 pc  
D. 560 pc  
E. 1150 pc

6. The Ba-α (rest wavelength = 656.28 nm) lines from two stars in an eclipsing binary system are observed over the course of one complete orbit of the binary system. The maximum (heliocentric) measured Doppler shifts are ±0.01 nm and ±0.03 nm about the equilibrium position, respectively. The period of the system is 10 yr. What are the masses of the two stars?

A. 6.6 $M_{\text{Sun}}$ and 13.3 $M_{\text{Sun}}$  
B. 0.57 $M_{\text{Sun}}$ and 1.72 $M_{\text{Sun}}$  
C. 6.6 $M_{\text{Sun}}$ and 20 $M_{\text{Sun}}$  
D. 0.08 $M_{\text{Sun}}$ and 0.24 $M_{\text{Sun}}$  
E. 1 $M_{\text{Sun}}$ and 3 $M_{\text{Sun}}$

7. At what wavelength does the continuous (black-body) spectrum from a sunspot at a temperature of 3800 K peak?

A. $7.63 \times 10^{-9}$ m  
B. 763 μm  
C. 7630 nm  
D. 7.6 μm  
E. 0.76 μm

8. What is the ratio of the intensity of radiation measured at a wavelength of 550 nm from a sunspot (3800K) and from the normal solar photosphere at 5800 K?

A. 0.18
9. Assume that the Sun radiates like a perfect spherical black body at a temperature of 5800 K. What is the total flux of radiation impacting on the Earth at 1 AU.

A. $1.8 \times 10^{17}$ J/s  
B. $7.1 \times 10^{17}$ J/s  
C. $3.6 \times 10^{17}$ J/s  
D. $2 \times 10^{24}$ J/s  
E. $8 \times 10^{24}$ J/s

10. If the albedo of a dust grain is independent of the wavelength of radiation, calculate the temperature of a dust grain 100 AU from an F0V star.

A. 40 K  
B. 75 K  
C. 58 K  
D. 60 °C  
E. 2.7 K

11. The material in our Galaxy is in an approximate circular orbit around the Galaxy’s centre of mass. If the rotational speed of the local standard of rest (8.5 kpc from the Galactic centre) is 220 km/s, what is the mass contained within the central 8.5 kpc of the Galaxy?

A. $10^{10}$ $\text{M}_\odot$  
B. $10^{11}$ $\text{M}_\odot$  
C. $10^5$ $\text{M}_\odot$  
D. $10^8$ $\text{M}_\odot$  
E. We don’t know because most of the mass is dark matter

12. A uniform density, spherical core in a molecular cloud is composed of $10^5$ hydrogen molecules / cm$^3$, estimate the minimum radius of this core if it was gravitationally bound at a temperature of 30 K.

A. 15 pc  
B. 8 pc  
C. $6.4 \times 10^{-3}$ pc  
D. 0.08 pc  
E. 0.15 pc
13. A uniform density, spherical core in a molecular cloud is composed of $10^5$ hydrogen molecules $/cm^3$, estimate the minimum mass of this core if it was gravitationally bound at a temperature of 30 K.

A. $10 \, M_{\text{Sun}}$
B. $18 \, M_{\text{Sun}}$
C. $40 \, M_{\text{Sun}}$
D. $0.06 \, M_{\text{Sun}}$
E. $400 \, M_{\text{Sun}}$

14. What is the escape velocity from the surface of the Sun?

A. $1.3 \times 10^8 \, \text{cm/s}$
B. $4.4 \times 10^7 \, \text{cm/s}$
C. $4.4 \times 10^9 \, \text{cm/s}$
D. $3 \times 10^{10} \, \text{cm/s}$
E. $6.2 \times 10^7 \, \text{cm/s}$

15. What is the Schwarzschild radius ($R_s$) of a black hole of mass $10^8 \, M_{\text{Sun}}$?

A. $5.4 \times 10^6 \, \text{m}$
B. $3 \times 10^{13} \, \text{cm}$
C. $5.4 \times 10^6 \, \text{cm}$
D. $3 \times 10^{13} \, \text{m}$
E. $1.5 \times 10^{13} \, \text{m}$
Part 2. Total 25%

2a. Estimate the mass loss rate \( \frac{dM}{dt} \) associated with the Sun’s current luminosity. Give your answer in solar mass per year. (3%)

2b. Give a brief account of the important processes occurring during the key stages in the post main sequence evolution of a star with a mass equal to the Sun. Illustrate your answer with a Hertzprung-Russell diagram. (10%)

2c. In a pure carbon white dwarf of one solar mass, what is the total number of particles? You may assume the carbon is fully ionised at a uniform temperature of \( 10^7 \) K. (3%)

2d. At this temperature estimate the total thermal energy of the white dwarf. (4%)

2e. A white dwarf is white because it is surrounded by a thin atmosphere of non-degenerate gas radiating a temperature of around \( 10^4 \) K. Its radius is around 100 times smaller than the Sun. Use this information to estimate the cooling lifetime of the white dwarf. (5%)
Part 3. Total 30%

3a. Give an example of a step-by-step reaction sequence which can convert *protons to alpha particles* in the centre of the Sun (spectral type G2V) at a temperature of around $10^7$ K. Highlight the rate-determining reaction step in the sequence. (6%)

3b. Starting from the equation for hydrostatic equilibrium,

$$\frac{dP}{dr} = -GM(r)\rho(r)/r^2$$

demonstrate that the pressure in the centre of a star is proportional to $M^2/R^4$, where $M$ is the mass of the star and $R$ is the radius. You may assume the star is a constant density uniform sphere. (10%)

3c. Hence, use an appropriate equation of state for a main sequence star to show that the temperature at the centre of a star is proportional to $M/R$. (6%)

3d. Using the answers to the questions above, together with the data for main sequence stars in Appendix 1, discuss why there are no main sequence stars with a mass less than about $0.1 M_{\text{Sun}}$. Illustrate your answer with an appropriate graph. (8%)
Appendix 1. Properties of main sequence stars

<table>
<thead>
<tr>
<th>Spectral type</th>
<th>$M_V$</th>
<th>$B-V$</th>
<th>$T_{\text{eff}}$(K)</th>
<th>$M/M_{\text{Sun}}$</th>
<th>$R/R_{\text{Sun}}$</th>
<th>$L/L_{\text{Sun}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O5</td>
<td>-6</td>
<td>-0.45</td>
<td>35000</td>
<td>39.8</td>
<td>17.8</td>
<td>3.2 x 10^5</td>
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<tr>
<td>B0</td>
<td>-3.7</td>
<td>-0.31</td>
<td>21000</td>
<td>17.0</td>
<td>7.6</td>
<td>1.3 x 10^4</td>
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<tr>
<td>B5</td>
<td>-0.9</td>
<td>-0.17</td>
<td>13500</td>
<td>7.1</td>
<td>4.0</td>
<td>6.3 x 10^2</td>
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<tr>
<td>A0</td>
<td>+0.7</td>
<td>+0.0</td>
<td>9700</td>
<td>3.6</td>
<td>2.6</td>
<td>7.9 x 10^1</td>
</tr>
<tr>
<td>A5</td>
<td>+2.0</td>
<td>+0.16</td>
<td>8100</td>
<td>2.2</td>
<td>1.8</td>
<td>2.0 x 10^1</td>
</tr>
<tr>
<td>F0</td>
<td>+2.8</td>
<td>+0.30</td>
<td>7200</td>
<td>1.8</td>
<td>1.4</td>
<td>6.3</td>
</tr>
<tr>
<td>F5</td>
<td>+3.8</td>
<td>+0.45</td>
<td>6500</td>
<td>1.4</td>
<td>1.2</td>
<td>2.5</td>
</tr>
<tr>
<td>G0</td>
<td>+4.6</td>
<td>+0.57</td>
<td>6000</td>
<td>1.1</td>
<td>1.05</td>
<td>1.3</td>
</tr>
<tr>
<td>G5</td>
<td>+5.2</td>
<td>+0.70</td>
<td>5400</td>
<td>0.9</td>
<td>0.93</td>
<td>7.9 x 10^1</td>
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<tr>
<td>K0</td>
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<td>+0.81</td>
<td>4700</td>
<td>0.8</td>
<td>0.85</td>
<td>4.0 x 10^1</td>
</tr>
<tr>
<td>K5</td>
<td>+7.4</td>
<td>+1.11</td>
<td>4000</td>
<td>0.7</td>
<td>0.74</td>
<td>1.6 x 10^1</td>
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<tr>
<td>M0</td>
<td>+8.9</td>
<td>+1.39</td>
<td>3300</td>
<td>0.5</td>
<td>0.63</td>
<td>6.3 x 10^2</td>
</tr>
<tr>
<td>M5</td>
<td>+12.0</td>
<td>+1.61</td>
<td>2600</td>
<td>0.2</td>
<td>0.32</td>
<td>7.9 x 10^3</td>
</tr>
</tbody>
</table>

(1) Absolute $V$-band magnitude  
(2) Effective surface temperature

Appendix 2. Physical constants

- Speed of light $c$: $2.998 \times 10^{10}$ cm s$^{-1}$, $2.998 \times 10^8$ m s$^{-1}$
- Gravitational constant $G$: $6.673 \times 10^{-8}$ dyne cm$^2$ g$^{-2}$, $6.673 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$
- Boltzmann constant $k$: $1.381 \times 10^{-16}$ erg K$^{-1}$, $1.381 \times 10^{-23}$ J K$^{-1}$
- Planck’s constant $h$: $6.626 \times 10^{-27}$ erg s, $6.626 \times 10^{-34}$ J s
- Stefan–Boltzmann constant $\sigma$: $5.670 \times 10^{-5}$ erg cm$^{-2}$ K$^{-4}$ s$^{-1}$, $5.670 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$
- Wien displacement constant $\lambda_{\text{max}}T$: $2.898 \times 10^{-1}$ cm K, $2.898 \times 10^{-3}$ m K
- Rydberg constant $R$: $1.097 \times 10^5$ cm$^{-1}$, $1.097 \times 10^7$ m$^{-1}$
- Mass of proton $m_p$: $1.6726 \times 10^{-24}$ g, $1.6726 \times 10^{-27}$ kg
- Mass of neutron $m_n$: $1.6749 \times 10^{-24}$ g, $1.6749 \times 10^{-27}$ kg
- Mass of electron $m_e$: $9.1096 \times 10^{-28}$ g, $9.1096 \times 10^{-31}$ kg
- Mass of hydrogen atom $m_H$: $1.6735 \times 10^{-24}$ g, $1.6735 \times 10^{-27}$ kg

Appendix 3. Astronomical constants

- Astronomical unit $AU$: $1.496 \times 10^{13}$ cm, $1.496 \times 10^{11}$ m
- Parsec $pc$: $3.086 \times 10^{18}$ cm, $3.086 \times 10^{16}$ m
- Solar mass $M_{\text{Sun}}$: $1.989 \times 10^{33}$ g, $1.989 \times 10^{30}$ kg
- Solar radius (mean) $R_{\text{Sun}}$: $6.960 \times 10^{10}$ cm, $6.960 \times 10^8$ m
- Solar luminosity $L_{\text{Sun}}$: $3.839 \times 10^{33}$ erg s$^{-1}$, $3.839 \times 10^{26}$ J s$^{-1}$
- Earth mass $M_E$: $5.977 \times 10^{27}$ g, $5.977 \times 10^{24}$ kg
- Earth radius (mean) $R_E$: $6.371 \times 10^8$ cm, $6.371 \times 10^6$ m
- Jupiter mass $M_J$: $1.899 \times 10^{30}$ g, $1.899 \times 10^{27}$ kg
- Jupiter radius (mean) $R_J$: $6.991 \times 10^9$ cm, $6.991 \times 10^7$ m
Appendix 4. The equations of stellar colour

Planck’s empirical law: Energy per second per frequency interval per unit area

\[ I(\nu, T) = \frac{2h\nu^3}{c^2} \left[ \exp\left(\frac{h\nu}{kT}\right) - 1 \right] \]

Planck’s empirical law: Energy per second per wavelength interval per unit area

\[ I(\lambda, T) = \frac{2hc^2}{\lambda^5} \left[ \exp\left(\frac{hc}{\lambda kT}\right) - 1 \right] \]

Wien’s displacement law: wavelength of maximum intensity

\[ \lambda_{\text{max}} T = 2.898 \times 10^6 \text{ nm K} \]

Stefan-Boltzmann law: Integrated energy per second per unit surface area

\[ E = \sigma T^4 \]

Integrated energy per second from a sphere: e.g. the total (bolometric) luminosity of a star

\[ L = 4\pi R^2 \sigma T^4 \]