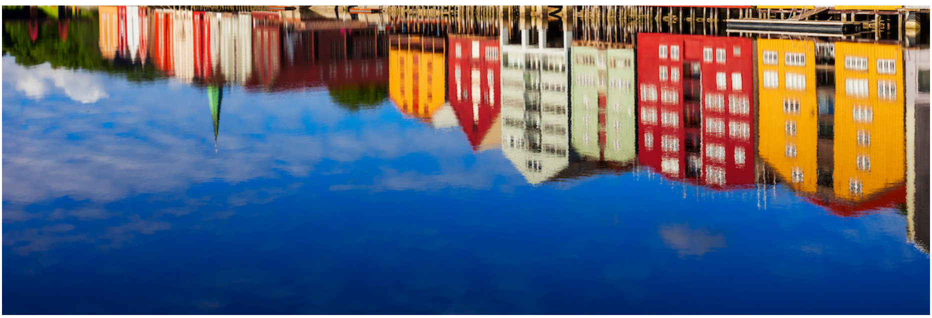
EXCURSIONS IN ANALYSIS

	Tuesday	Wednesday	Thursday	Friday
08:30	*Registration*			
09:00	Harper	Kupiainen	Hartmann	Radziwiłł
10:00	Webb	Bayart	Aleman	Sodin
11:00	Aistleitner	Carneiro	Tylli	Hedenmalm
12:00	*Lunch*	*Lunch*	*Lunch*	*Lunch*
13:30	Radchenko	Ortega-Cerdà	Bonk	Bondarenko
14:30	Nitzan		Koskela	Poltoratski
15:30	Lyubarskii		Lassas	Queffélec
19:00			*Dinner*	



The distribution of short moving character sums

Speaker. Adam Harper (University of Warwick).

Abstract. Sums of Dirichlet characters are one of the most studied objects in analytic number theory. In this talk I will describe some work on the distribution of

$$\sum_{x<n\leq x+H} \chi(n),$$

where χ is a non-principal character mod q and x varies between 0 and $q-1$. This problem was investigated by Davenport and Erdős, and more recently by Lamzouri and others. Lamzouri conjectured that provided $H \to \infty$ but $H = o(q/\log q)$, the sum should have a Gaussian limiting distribution. I will present some results that shed more light on this conjecture.

Is complex analysis (mathematical) physics?

Speaker. Christian Webb (University of Helsinki).

Abstract. I will discuss a classical result in complex analysis relating various kernel functions on planar domains, and how from the point of view of mathematical physics, this result is an instance of a fundamental fact known as bosonization. Time permitting, I will discuss some joint work with B. Bayraktoroglu, K. Izyurov, and T. Virtanen, about applications of bosonization to the critical Ising model.

My encounter with Kristian: GCD sums and the zeta function

Speaker. Christoph Aistleitner (TU Graz).

Abstract. I will recount how I came into contact with Kristian in 2012, and how this eventually led to the publication of two joint papers on GCD sums, spectral norms of GCD matrices, and convergence criteria for series of dilated functions (together with the other co-authors Istvan Berkes and Michel Weber). I will explain how the questions we studied arose from classical problems in equidistribution theory, metric number theory, and harmonic analysis, and how quite surprisingly our work turned out to be closely connected to the problem of finding large values of the Riemann zeta function in the critical strip, and to provide a universal framework for metric pair correlation problems that had been studied earlier by Rudnick and Sarnak.

Fourier interpolation, uniqueness, and energy minimization problems

Speaker. Danylo Radchenko (ETH Zurich).

Abstract. I will discuss some recently discovered formulas that allow one to reconstruct any sufficiently nice function from discrete samples of its values and the values of its Fourier transform. Somewhat surprisingly, the construction of these interpolation formulas involves weakly holomorphic modular forms for the principal congruence subgroup of level 2 in $\mathrm{PSL}_2(\mathbb{Z})$. I will also discuss the connection of these results to the energy minimization problem in Euclidean spaces and some related open questions.

On Riesz bases and Frames of Exponentials

Speaker. Shahaf Nitzan (Georgia Tech).

Abstract. I will survey a few results related to the talks title and discuss the recent construction, joint with G. Kozma and A.Olevskii, of a bounded set that does not admit a Riesz basis of exponentials.

Gabor frames from rational functions

Speaker. Yurii Lyubarskii (NTNU).

Abstract. Let g be a function in $L^2(\mathbb{R})$. By G_Λ, $\Lambda \subset R^2$ we denote the system of time-frequency shifts of g, $G_\Lambda = \{e^{2\pi i\omega x} g(x-t)\}_{(t,\omega)\in\Lambda}$.

A typical model set Λ is the rectangular lattice $\Lambda_{\alpha,\beta} := \alpha\mathbb{Z} \times \beta\mathbb{Z}$ and one of the basic problems of the Gabor analysis is the description of the frame set of g i.e., all pairs α, β such that $G_{\Lambda_{\alpha,\beta}}$ is a frame in $L^2(\mathbb{R})$.

It follows from the general theory that $\alpha\beta \leq 1$ is a necessary condition (we assume $\alpha, \beta > 0$, of course). Do all such α, β belong to the frame set of g?

Up to 2011 only few such functions g (up to translation, modulation, dilation and Fourier transform) were known. In 2011 K. Grochenig and J. Stockler extended this class by including the totally positive functions of finite type (uncountable family yet depending on finite number of parameters) and later added the Gaussian finite type totally positive functions.

We suggest another approach to the problem and prove that all Herglotz rational functions with imaginary poles also belong to this class. This approach also gives new results for general rational functions.

Solving Liouville conformal field theory

Speaker. Antti Kupiainen (University of Helsinki).

Abstract. Liouville conformal field theory (LCFT) can be viewed as a quantum or probabilistic extension of the classical story of uniformisation of Riemann surfaces. To each Riemann surface with marked points LCFT assigns a probabilistic object, "correlation function" which is naturally defined on the moduli space of such surfaces. I will review recent work where closed formuli for these functions are proved thereby realising the so called "conformal bootstrap" program in physics.

Joint work with C. Guillarmou, R. Rhodes and V. Vargas.

Universality with respect to translations of general Dirichlet series

Speaker. Fréderic Bayart (University Clermont Auvergne).

Abstract. A seminal result of Voronin says that the Riemann Zeta function is universal with respect to translations in the critical strip: for any compact subset K with connected complement contained in $\{s: \Re e(s) \in (1/2,1)\}$, for any function f which is continuous on K, which is holomorphic inside K and which does not vanish, there is a sequence of vertical translates of the Zeta function which converges uniformly to f on K. Since then, many other examples of Dirichlet series have been proved to be universal, like L-functions or Lerch Zeta functions. In this talk, we are interested in general Dirichlet series $\sum_{n=1}^{+\infty} a_n e^{-\lambda_n s}$. We give sufficient conditions on (a_n) and (λ_n) such that the Dirichlet series is universal in some adequate strip, or half-plane.

Some Fourier optimization problems in number theory

Speaker. Emanuel Carneiro (ICTP).

Abstract. This will be a talk on how some Fourier optimization problems, part of them related to reproducing kernel Hilbert spaces, appear in connection to problems in number theory. More specifically we will discuss recent new bounds related to prime gaps, to the pair correlation conjecture, and to low-lying zeros of families of L-functions.

Contractive projections in H^p spaces

Speaker. Joaquim Ortega-Cerdà (University of Barcelona).

Abstract. I will present a joint work with Ole Fredrik Brevig and Kristian Seip. We describe the idempotent Fourier multipliers that act contractively on H^p spaces of the d-dimensional torus $\mathbb{T}^d$ for $d \geq 1$ and $1 \leq p \leq \infty$. When p is not an even integer, such multipliers are just restrictions of contractive idempotent multipliers on L^p spaces, which in turn can be described by suitably combining results of Rudin and Andô. When $p = 2(n+1)$, with n a positive integer, contractivity depends in an interesting geometric way on n, d, and the dimension of the set of frequencies associated with the multiplier. Our results allow us to construct a linear operator that is densely defined on $H^p(\mathbb{T}^\infty)$ for every $1 \leq p \leq \infty$ and that extends to a bounded operator if and only if $p = 2, 4, \ldots, 2(n+1)$.

Multiple interpolation and sampling in spaces of holomorphic functions

Speaker. Andreas Hartmann (University of Bordeaux).

Abstract. Interpolating and sampling sequences have been studied in many spaces of analytic functions and play an important role in numerous applications and domains in mathematics such as signal theory, control theory, operator theory, etc. Classically, one is interested in getting information on a function from specific values in given points.

A natural question is to look not only at the values of a function but also at its derivatives up to a certain order in the interpolation or sampling nodes. This leads to so-called multiple interpolation and sampling. Morally speaking, for a sampling problem for instance, increasing the information in a node should allow for decreasing the number of nodes.

Multiple interpolation problems are completely understood in the Hardy space thanks to work by Nikolski, Vasyunin, Volberg and others starting in the late 70's. In the Fock space, Seip and his collaborators were investigating in the 90's first classical interpolation and sampling with multiplicity one, and, later on, also the case with uniformly bounded multiplicities. In both cases, sampling and interpolation properties are characterized by density conditions which are incompatible, thereby implying that there are no simultaneous sampling and interpolating sequences (Riesz bases). Seip and Brekke then asked whether there might exist such sequences in case the multiplicites tend to $+\infty$ fast enough.

In 2017, in joint work with Borichev, Kellay and Massaneda, we were investigating sampling and interpolation properties in Fock spaces with arbitrary multiplicities for which we identified necessary as well as sufficient conditions based on covering and separation conditions. While these conditions do not characterize interpolation or sampling, they are sufficiently sharp to give a negative answer to the Seip–Brekke question: whenever the multiplicities tend to $+\infty$, there are no simultaneous sampling and interpolating sequences. More recently, with D. Aadi, C. Cruz and K. Kellay, we were considering the situation in Bergman spaces for which the underlying geometry is more intricate. Though necessary and sufficient conditions are expressed in a similar way, understanding for instance the critical radius for the covering and separation conditions is a delicate problem. In this talk I will present and explain some of the above mentioned results.

Composition of analytic paraproducts

Speaker. Alexandru Aleman (Lund University).

Abstract. For a fixed analytic function g in the unit disc, we consider the analytic paraproducts induced by g, which are defined by

$$T_g f(z) = \int_0^z f(\zeta) g'(\zeta)\, d\zeta, \qquad S_g f(z) = \int_0^z f'(\zeta) g(\zeta)\, d\zeta,$$

together with the multiplication operator $M_g f(z) = f(z)g(z)$. The boundedness of these operators on various spaces of analytic functions on the unit disc is well understood. The original motivation for this work is to understand the boundedness of compositions (products) of two of these operators, for example T_g^2, $T_g S_g$, $M_g T_g$, etc. The talk intends to present a general approach which yields a characterization of the boundedness of a large class of operators contained in the algebra generated by these analytic paraproducts acting on the classical weighted Bergman and Hardy spaces in terms of the symbol g. In some cases it turns out that this property is not affected by cancellation, while in others it requires stronger and more subtle restrictions on the oscillation of the symbol g than the case of a single paraproduct.

This is a report about joint work with C. Cascante, J. Fàbrega, D. Pascua and J. A. Peláez.

Rigidity of some classical operators on Hardy spaces

Speaker. Hans-Olav Tylli (University of Helsinki).

Abstract. I will describe linear qualitative properties of non-compact composition operators C_ϕ and non-compact analytic Volterra operators T_g on the Hardy spaces H^p over the unit disk $\mathbb{D} = \{z : |z| < 1\}$ for $p \neq 2$, as well as the motivation. Here

$$f \mapsto C_\phi(f) = f \circ \phi, \qquad f \mapsto T_g f(z) = \int_0^z f(w)g'(w)dw,$$

where the analytic maps $\phi : \mathbb{D} \to \mathbb{D}$, respectively $g \in BMOA$, are the given symbols. The main results imply that non-compact C_ϕ and T_g display a quite restricted qualitative behaviour compared to that of arbitrary bounded operators on H^p for $1 < p < \infty$ and $p \neq 2$. For instance, if T_g defines an isomorphism $M \to T_g(M)$ when restricted to an infinite-dimensional closed subspace $M \subset H^p$, then M contains a subspace linearly isomorphic to ℓ^p. The analogous behaviour in the class of C_ϕ is more complicated, but still determined by the restrictions to subspaces $M \subset H^p$ which are linearly isomorphic to ℓ^p or ℓ^2. By contrast, compact operators C_ϕ or T_g show a rich variety of behaviour.

The talk revisits joint works with Jussi Laitila (Helsinki), Santeri Miihkinen (Abo), Pekka Nieminen (Turku) and Eero Saksman (Helsinki).

Thurston maps and the dynamics on curves

Speaker. Mario Bonk (UCLA).

Abstract. A lot of William Thurston's work can be interpreted from the desire to characterize conformal dynamical systems (such as Kleinian groups or rational maps) among a larger class of dynamical systems defined by geometric-topological conditions. A famous result in this direction is his characterization of postcritically-finite rational maps among what are now called Thurston maps.

One of Thurston's key insight is that rationality of a Thurston map is closely related to dynamical properties of simple closed curves under the pull-back operation by the map. In my talk I will give a gentle introduction to some of these themes and focus on a stubborn open problem in this area: the existence of a finite global curve attractor for postcritically-finite rational maps.

Quasiconformal Hardy classes

Speaker. Pekka Koskela (University of Jyväskylä).

Abstract. According to the classical definition of a Hardy space, H^p, $0 < p < \infty$, is the class of those analytic functions f for which the integrals of $|f|^p$ over circles of radii r remain bounded when r increases to 1. Given a simply connected domain Ω, the Hardy number of Ω is then the supremum of the exponents $p > 0$ for which f belongs to H^p for a fixed conformal map (analytic and univalent) from the unit disk onto Ω. This is independent of the choice of the conformal map. We describe attempts to extend these concepts to the class of quasiconformal maps.

This is based on joint work with Ondra Bouchala.

Inverse problems in analysis and probability

Speaker. Matti Lassas (University of Helsinki).

Abstract. In many deterministic inverse problems one aims to determine a function from its integrals over a family of curves or the coefficient functions of a partial differential equation from boundary measurements. These problems lead to statistical inverse problems where the unknown objects are modelled by random functions and the measurements contain random errors. We give a review on methods to solve these inverse problems. In particular, we consider X-ray tomography in different applications.

Gaps between $\sqrt{n} \mod 1$

Speaker. Maksym Radziwiłł (Caltech).

Abstract. In the 2000's Elkies and McMullen showed that the gap distribution of the sequence $\sqrt{n} \mod 1$ exists and is not Poisson. This is a unique behavior, as for all non-integer positive α different from 1/2 it is conjectured that the gap distribution of $n^\alpha \mod 1$ is Poisson. Elkies's and McMullen's proof used among other things ideas from dynamics and Teichmuller theory. In a recent work with Niclas Technau we revisit this problem and provide a purely analytic, one could even say elementary, proof. I will discuss some of the ideas involved.

Distribution of zeroes of entire functions and binary correlations of Taylor coefficients

Speaker. Misha Sodin (Tel Aviv University).

Abstract. The topic of my talk will be a remarkable connection between spectral properties of Taylor coefficients of entire functions and their zero-distribution on different scales. This connection works in many different instances of random and pseudo-random coefficients (among them are Besicovitch almost periodic sequences, random stationary sequences, multiplicative random sequences, arithmetic sequences of Diophantine nature, the indicator function of the square free integers, and many others). The talk is based on joint works with Jacques Benatar, Alexander Borichev and Alon Nishry.

Soft Riemann–Hilbert problems and planar orthogonal polynomials

Speaker. Håkan Hedenmalm (KTH Stockholm).

Abstract. Riemann–Hilbert problems are fundamental jump problems. Here, the jump is soft which is why it was not understood how to solve it until recently. The work improves on earlier work with A. Wennman.

Fourier interpolation with zeroes of zeta

Speaker. Andriy Bondarenko (NTNU).

Abstract. We will discuss our recent construction of a Fourier interpolation basis for functions analytic in a strip symmetric about the real line involving non-trivial zeros of ζ.

Convergence of scattering data

Speaker. Alexei Poltoratski (University of Wisconsin).

Abstract. The scattering transform for the Dirac system of differential equations is commonly viewed as a non-linear version of the classical Fourier transform. This connection leads to natural questions on finding analogs of various properties of the Fourier transform in non-linear settings. In my talk I will give a short overview of that area and present a version of Carleson's theorem on pointwise convergence in the non-linear case.

On the Furstenberg set and its random brother

Speaker. Hervé Queffélec (University of Lille).

Abstract. The purpose of this talk is the study of the (innocent-looking) Furstenberg set S of positive integers defined as

$$S = \{2^j 3^k\}_{j,k\geq 0} =: \{s_1 < s_2 < \cdots s_n < \cdots\}.$$

Note that (s_n) is subexponential and superpolynomial. S will be studied from three points of view:

1. Number theory and arithmetic: e.g. asymptotic behavior of
$$s_n, \qquad s_{n+1} - s_n, \qquad s_{n+1}/s_n,$$
based on irrationality measures of the transcendental number $\alpha = \log 2/\log 3$.
2. Dynamics: e.g. distribution mod. 1 of $(s_n x)$ for $x \in (0,1)$.
3. Harmonic analysis: is S a $\Lambda(q)$-set, a Sidon set, a p-Sidon set, for some $2 < q < \infty,\ 1 < p < 2$?

The same questions will be posed with a random analogue T of S, defined in the Erdös-Rényi fashion:

$$T = T(\omega) = \{k \geq 1 : \xi_k(\omega) = 1\}$$

where (ξ_k) is a sequence of independent, $0-1$-valued random variables with $\mathbb{E}(\xi_k) = \delta_k$, to be adjusted.

A conjecture of Furstenberg, far beyond our scope, but related, acted as a motivation for the study of S and T.

This is joint work (partly a survey) with A. H. Fan and M. Queffélec.