Load-Strength Interference

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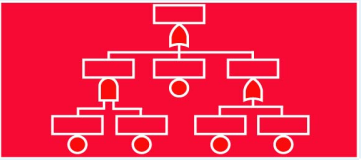
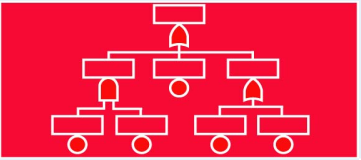


Load and strength

A component will fail when:

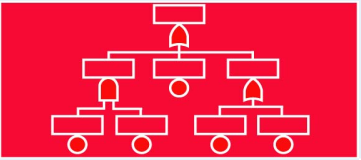
$$\text{Load (or stress)} > \text{Strength}$$

- Load: mechanical, electrical, thermal, chemical – environmental or operating
- Strength (resisting property), e.g., strength, hardness, melting point.

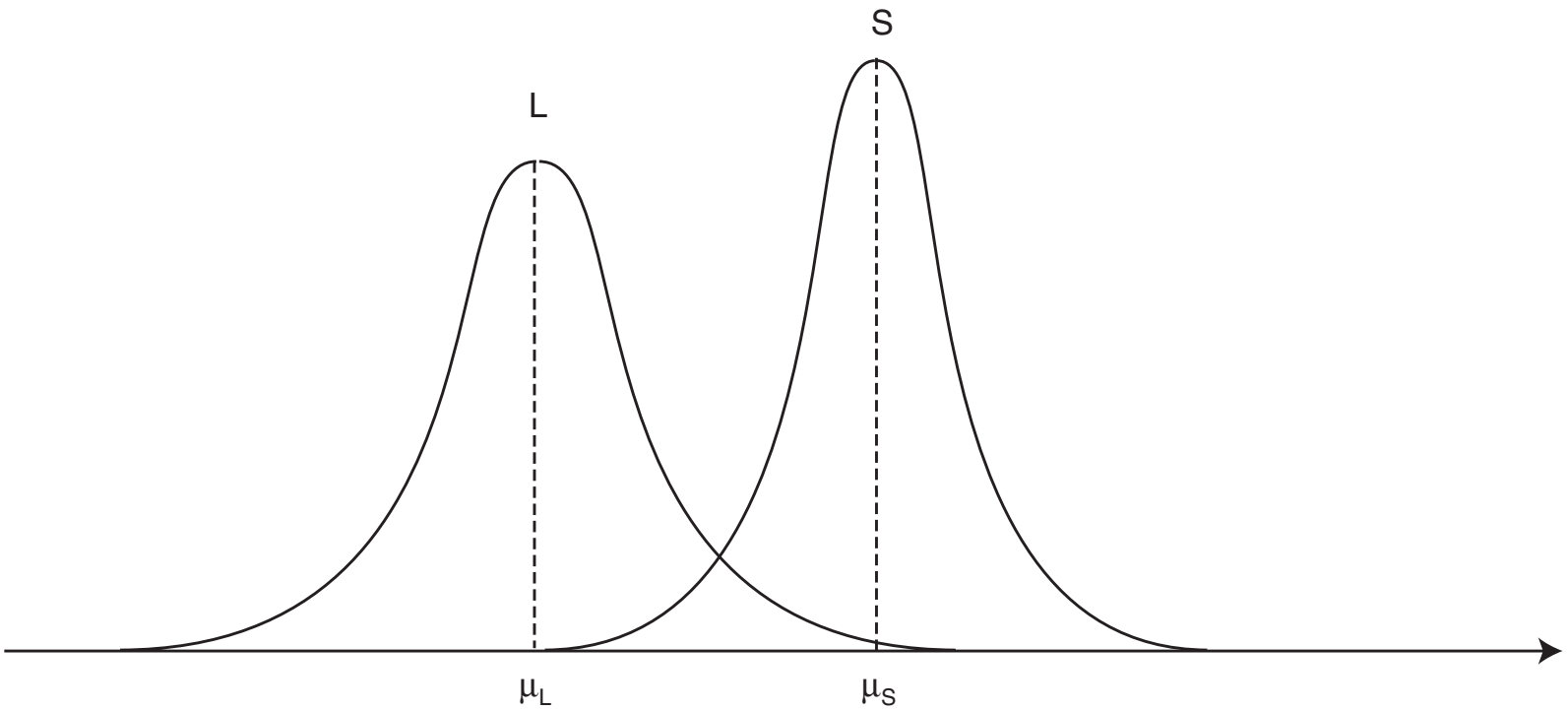


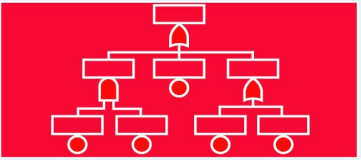
Examples of load-strength failures

- Shaft: Fractures when torque exceeds strength
- Bearing: Internal generated loads exceed the local strength, causing fracture, overheating, etc.
- Electric load higher than capacity (strength)
- No of telephone calls higher than capacity ("strength") of telecommunication system



Load-strength interference





Safety factor

The safety factor is sometimes defined as

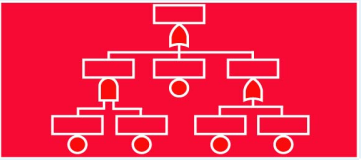
$$\mathrm{SF} = \frac{\text{“Strength of the system”}}{\text{“Load placed on the system”}}$$

or, perhaps

$$\mathrm{ST} = \frac{\mu_S}{\mu_L}$$

or

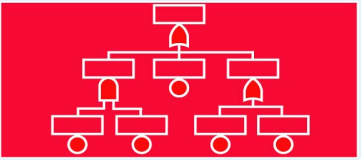
$$\mathrm{SF} = \frac{\mathrm{Mode}(S)}{\mathrm{Mode}(L)}$$



Safety margin

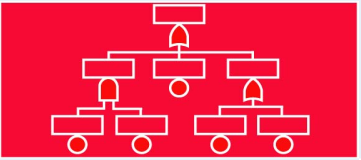
The safety margin (SM) is the "normalized" difference between the system strength and the load

$$\mathrm{SM} = \frac{\mu_S - \mu_L}{\sqrt{\sigma_S^2 + \sigma_L^2}}$$



Loading roughness

$$\mathrm{LR} = \frac{\sigma_L}{\sqrt{\sigma_S^2 + \sigma_L^2}}$$



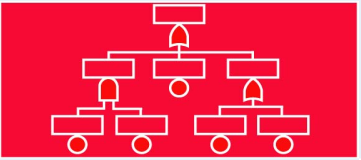
Single load application

We assume that the strength (S) and the load (L) may be regarded as independent, positive variables. The reliability of the item is then

$$\begin{aligned} R = \Pr(S > L) &= \int_0^\infty \Pr(S > l) f_L(l)\, dl \\ &= \int_0^\infty \left(\int_l^\infty f_S(s)\, ds \right) dl \end{aligned}$$

We may also use

$$\begin{aligned} R = \Pr(L < S) &= \int_0^\infty \Pr(L < s) f_S(s)\, ds \\ &= \int_0^\infty \left(\int_0^s f_L(l)\, dl \right) ds \end{aligned}$$



Single load application (Example)

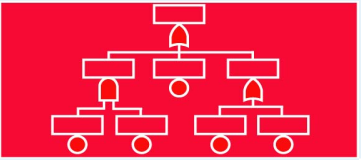
When $S \sim \mathcal{N}(\mu_S, \sigma_S^2)$ and $L \sim \mathcal{N}(\mu_L, \sigma_l^2)$, we have

$$S - L \sim \mathcal{N}(\mu_S - \mu_L, \sigma_S^2 + \sigma_L^2)$$

and

$$R = \Pr(S - L > 0) = \Phi\left(\frac{\mu_S - \mu_L}{\sqrt{\sigma_S^2 + \sigma_L^2}}\right) = \Phi(\mathrm{SM})$$

Note that the reliability, R, is a function of the safety margin, SM, in this case.



Other distributions

For some other combinations of distributions we get:

- When $S \sim \exp(\lambda_s)$ and $L \sim \exp(\lambda_L)$

$$R = \frac{\lambda_L}{\lambda_S + \lambda_L}$$

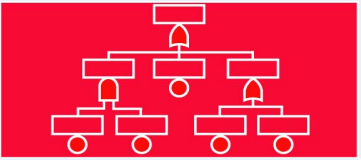
- When $S \sim \exp(\lambda_S)$ and $L \sim \mathcal{N}(\mu, \sigma^2)$

$$R = e^{-\frac{1}{2}(2\mu\lambda_S - \sigma^2\lambda_S^2)}$$

- When $S \sim \mathcal{N}(\mu, \sigma^2)$ and $L \sim \exp(\lambda_L)$

$$R = 1 - e^{-\frac{1}{2}(2\mu\lambda_L) - \sigma^2\lambda_L^2)}$$

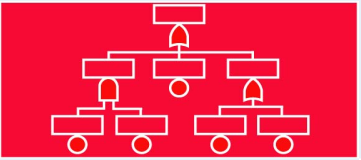
These distributions may, however, not be very realistic.



Repetitive loading

- Assume that the strength and the load are independent, and repeated n independent times, that is, for each repetition the strength (S) and the load (L) are chosen at random from the distributions $f_S(s)$ and $f_L(l)$, respectively. The reliability of the item is:

$$R_n = R_1 \cdot R_1 \cdots R_1 = R_1^n = (\Pr(S > L))^n$$



Repetitive loading (cont.)

- Assume now that we have one component with strength S that is exposed to a sequence of independent loads $L_1, L_2, \ldots, L_n$. The reliability of the item is:

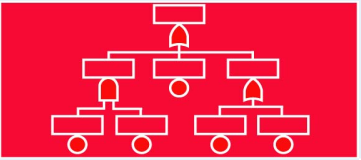
$$R_n = \Pr(\max_{i=1,2,\ldots,n}\{L_i\} < S)$$

The distribution of $\max\{L_i\}$ is

$$\Pr(\max\{L_i\} \leq l) = \Pr\left(\bigcap_{i=1}^{n} L_i \leq l\right) = \prod_{i=1}^{n} \Pr(L_i \leq l)$$

The reliability is hence

$$\begin{aligned} R_n = \Pr(\max\{L_i\} < S) &= \int_0^\infty \Pr(\max\{L_i\} < s) f_S(s)\, ds \\ &= \int_0^\infty \left(\prod_{i=1}^{n} \Pr(L_i \leq s)\right) f_S(s)\, ds \end{aligned}$$



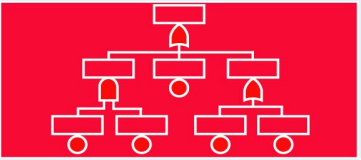
Repetitive loading (cont.)

The expression on the previous slide may also be written:

$$R_n = \int_0^\infty \left(\prod_{i=1}^{n} \left(\int_0^s f_{L_i}(l)\, dl \right) \right) f_S(s)\, ds$$

When the loads $L_1, L_2, \ldots, L_n$ have the same distribution $f_L(t)$ this expression becomes:

$$R_n = \int_0^\infty \left(\int_0^s f_L(l)\, dl \right)^n f_S(s)\, ds$$



Time dependent variables

In practice, both the strength (S) and the load (L) will usually be functions of time t. The reliability is then

$$R(t) = \Pr(S(t) > L(t)) = \Pr(S(t) - L(t) > 0)$$

May be extended some time in the future...