Harmonic Analysis and Comparison of the Back EMFs of Four Permanent Magnet Machines with Different Winding Arrangements

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***Abstract*-This paper investigates the back EMFs of four surface-mounted permanent magnet machines with different winding arrangements for education purpose. They are *a*): a 4-pole machine with integral-slot distributed winding, *b*): an 8-pole machine with fractional-slot distributed winding, *c)*: a 12-pole machine with integral-slot concentrated winding and *d*): a 24-pole machine with fractional-slot concentrated winding. The harmonic analysis of the back EMFs with different windings is firstly carried out by finite element analysis (FEA) in the software of Comsol. Furthermore, the harmonic comparisons among machines are preformed. Finally, the prototype machines of *a, b* and *d* are made in the laboratory for students to study different winding arrangements. Some measurements are also carried out to verify the FEA results.**

I. Introduction

The essential feature of an electrical machine is the armature winding in which the back EMF is induced. The layout of an armature winding can significantly influence the back EMF shape and hence affect the performances of the electrical machine. Basically, the harmonic components in back EMFs can effectively be reduced by selecting appropriate winding types and coil pitches. The winding type of an electrical machine is mainly determined by the number of q, slot per pole per phase, of the machine following the below regulars:

- Integral $q > 1$, integral-slot distributed winding.
- $q = 1$, integral-slot concentrated winding.
- Fractional $q > 1$, fractional-slot distributed winding.
- Fractional $q < 1$, fractional-slot concentrated winding.

In general, distributed windings can make better use of the stator and rotor structure and also decrease harmonics comparing to concentrated windings [1]. On the other hand, concentrated windings can reduce the volume of copper used in the end-windings as well as the copper losses [5]. Fractional-slot windings have advantage of reducing slot harmonics comparing to integral-slot windings [2]. And for high pole machines where high slot number is required for conventional integral-slot windings, fractional-slot windings need less stator slot and the copper volume in the stator cross-section therefore increases, by which the machine performance is improved [5][6]. Some combinations of slot number and pole number with fractional-slot winding will lead to low fundamental winding factors and have to avoid at the stage of machine design. [6].

By selecting coil pitch of the windings, certain harmonics in the phase EMFs can be further reduced or suppressed. For the purpose to reduce the amount of copper and copper losses in the end-connections of windings, it is usual to choose short coil pitches for most machines [2]-[4].

In order to systematically study how different winding types and coil pitches influence the back EMFs, four surface-mounted permanent magnet (PM) machines are investigated by finite element analysis (FEA). They are *a)*: 4-pole machine with integral-slot distributed winding, *b)*: 8-pole machine with fractional-slot distributed winding, *c)*: 12-pole machine with integral-slot concentrated winding and *d)*: 24-pole machine with fractional-slot concentrated winding. The harmonics in the EMFs of the machines with different coil pitches are investigated by FEA computations. And then the harmonic comparisons among the machines are performed. Finally, the machines *a*, *b* and *d* are made in the laboratory for students to study different winding arrangements. The prototype machines consist of a stator with 36 slots and a rotor with 24 small pieces of rectangular magnet on the rotor surface. They have open stators with re-connectable end windings as shown in Fig. 1 so that the winding can freely be re-arranged by varying the end winding connections.

Fig. 1. Open-stator machine with re-connectable end windings.

II. THEORETICAL LITERATURE

The airgap flux density distribution produced by full-pitch rectangular magnets in a surface-mounted PM machines, neglecting the slot effect and the magnetic flux leakage between the magnets, is ideally a square wave consisting of all odd harmonics as shown in Fig. 2. It can be expressed in Fourier series as (1) [1].

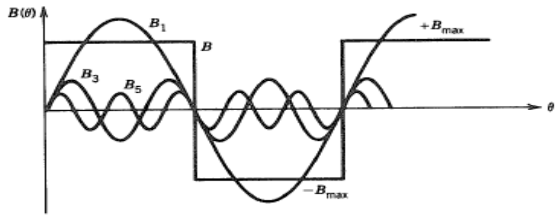


Fig. 2. Flux density distribution along airgap

$$B(\theta) = \sum_{h=1,3,5,...}^{\infty} \frac{4B_{\max}}{\pi h}\sin(h\theta) = \sum_{h=1,3,5,...}^{\infty} B_h \quad (1)$$

where h is harmonic order, B_h is the h order component of flux density, B_{max} is the maximum flux density of the square waveform.

According to Faraday's law, all the odd harmonics also present in the induced back EMF of a full-pitch turn in an electrical machine when the flux rotates, and the induced EMF can be expressed as:

$$e = -\frac{d\Phi}{dt} = \sum_{1,3,5,...}^{\infty} e_h \quad (2)$$

where e is the induced EMF in a full-pitch turn, Φ is the magnetic flux, e_h is the induced EMF component by the h order flux.

For an electrical machine with N-turn coil windings per phase, the resultant phase EMF is determined by (3).

$$e_p = N \sum_{h=1,3,5...}^{\infty} \xi_h e_h \quad (3)$$

where e_p is the phase EMF, ξ_h is the winding factor of the h order harmonic and is determined by (4).

$$\xi_h = k_{dh} \cdot k_{ph} \quad (4)$$

where k_{dh} and k_{ph} are distribution factor and pitch factor of the h order harmonic, respectively. And they are calculated by:

$$k_{dh} = \left(\sin(\frac{h\pi}{2m})\right) / \left(nq\sin(\frac{h\pi}{2mnq})\right) \quad (5)$$

$$k_{ph} = \sin\left(h\frac{W}{\tau}\cdot\frac{\pi}{2}\right) \quad (6)$$

where m is the phase number, τ is the pole pitch, W is the coil pitch expressed in term of pole pitch; q and n, for integral-slot windings, are respectively an integer and unity, and for fractional-slot windings they have the following relation:

$$q = \frac{z}{n} \quad (7)$$

where both z and n are integers.

The amount of distortion in an EMF is quantified by total harmonic distortion (THD), which is defined by

$$THD\% = 100 \cdot \left(\sqrt{\sum_{h=2}^{\infty} e_h^2}\right) / e_1 \quad (8)$$

From (1) and (2), we know that the harmonic component of e_h is inversely proportional to the harmonic order h, so the distortion is mainly determined by the low harmonics. This paper only considers the harmonic orders less than 100 for calculations whilst only the orders less than 20 are presented in figures about harmonic analysis.

III. MACHINE PARAMETERS

The rotor of each investigated machine consists of 24 small pieces of rectangular magnet on the surface. By varying the magnet configurations, the pole number of the machines can be easily changed so that different values of q are achieved. The possible pole numbers, p, of the machines are 2, 4, 6, 8, 12 and 24. Here only considering 3-phase two-layer windings, these machines respectively have q values of 6, 3, 2, 3/2, 1 and ½, and their distribution factors obtained from (5) are presented in Fig. 3. According to the previously mentioned regulars, the 2-, 4- and 6-pole machines with integral q greater than unity have integral-slot distributed windings, so only one of them, the 4-pole machine, is chosen together with the 8-, 12- & 24 machines for FEA computations. The reason to select the 4-pole machine than others is that it has the same distribution factors as the 8-pole machine as depicted in Fig. 3. If they have same coil pitches, their winding factors are same according to (3)-(7). So do the 12- & 24-pole machines. It is authors' interest to investigate the harmonic content in the EMFs with same winding factors, but different winding types. Fig. 4 shows their magnet configurations of the investigated machines and

TABLE **I** lists some their parameters. Here only the coil pitches from around 0.5 to 1 pole pitch are considered. In this paper, the coil pitches are expressed in term of pole pitch without extra notation.

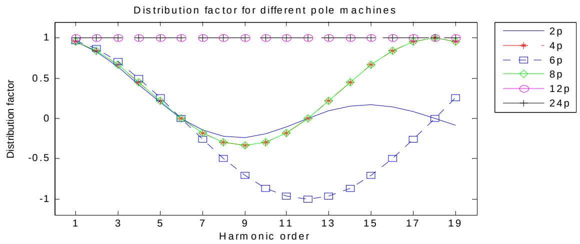
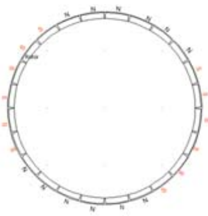
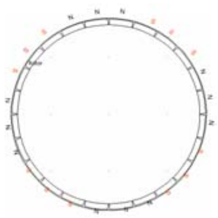


Fig. 3. Distribution factors of different pole machines


(a)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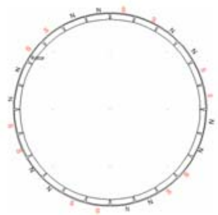
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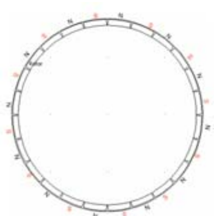

(d)

Fig. 4. Magnet configurations of (a) the 4-pole machine, (b) the 8-pole machine, (c) the 12-pole machine (d) the 24-pole machine.

TABLE I.

MACHINE PARAMETERS

p	4	8	12	24
q	3	1.5	1	0.5
n	1	2	1	2
W	9/9,8/9,7/9,6/9, 5/9,4/9	4/4.5, 3/4.5, 2/4.5	3/3,2/3,1/3	1/1.5

IV. FEA COMPUTATIONS

A. *EMF of a Full-Pitch Turn*

To study the affect of different windings to the EMFs, the induced EMF of a full-pitch turn for each machine should be known as reference. Normally it is assumed to be a square waveform for surface-mounted PM machines having one magnet per pole, but here the 4-, 8- & 12-pole machines respectively have 6, 3 & 2 magnets per pole. Due to the additional airgaps between the magnets, the magnetic flux density between the rotor and stator will slightly fluctuate and hence introduce extra harmonics. How this will influence the EMF is checked by investigating the 4-pole machine as shown in Fig. 5 and Fig. 6. The red dash line in Fig. 5 is the EMF waveform of a full-pith turn when the 4-pole machine has 6 magnets per pole, and the blue one is when it has one magnet per pole. In order to conveniently compare with the phase EMFs of the machines later, the values in Fig. 5 are scaled for 12 turns (total turn per phase). The harmonics of the waveforms are depicted in Fig. 6. The low-order harmonics just slightly changed and the additional harmonic is less than 9% of the THD. For simplification, we assume that the 8-, 12-& 24-pole machines approximately have the same as the blue waveform in a full-pitch turn, and then its harmonics in Fig. 6 can be used as reference to compare with the phase EMFs of the machines later. The assumption will not significantly influence the analysis and results presented in this paper.

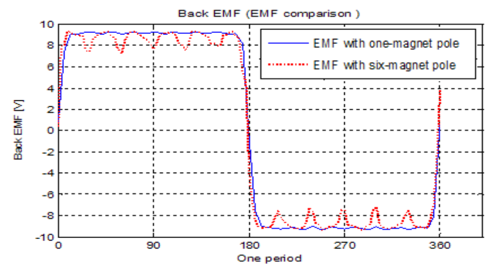


Fig. 5. Comparison of EMF waveforms of full pitch turns of the 4-pole machine with one magnet per pole and six magnets per pole.

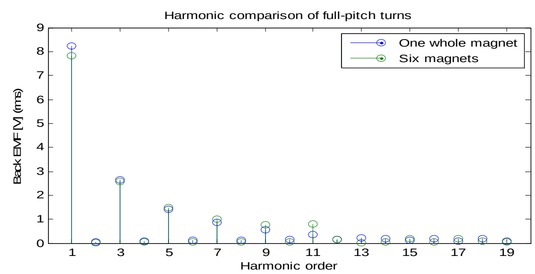
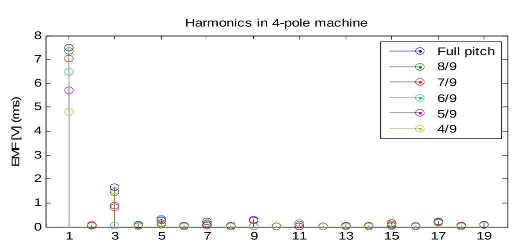


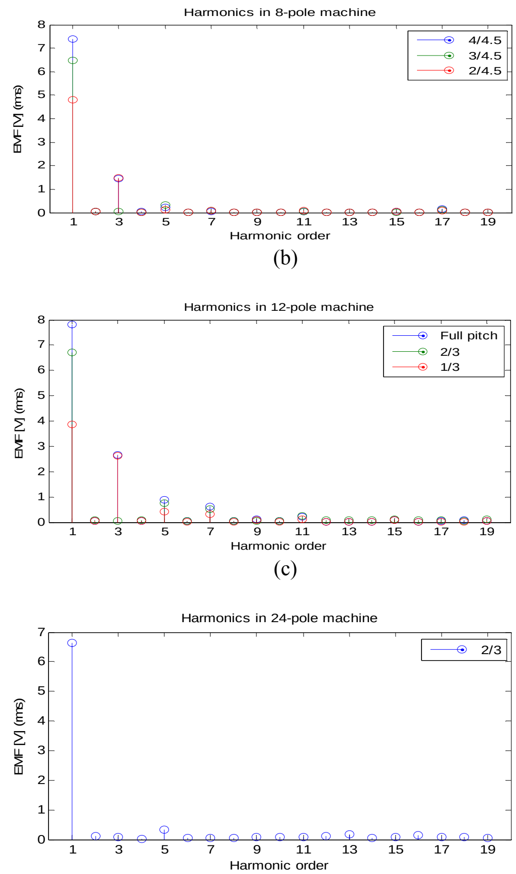
Fig. 6. Harmonic components of the EMFs in Fig. 5.

B. *Harmonics in Phase EMFs*

Harmonic analysis in the phase EMFs of the machines with various coil pitches is carried out and the results are presented in Fig. 7, individually. Comparing these harmonic components to the reference in Fig. 6, the harmonic reductions by the different winding types and coil pitches are clearly shown.



(a)



(b)

(c)

(d)

Fig. 7. Harmonic components obtained from the FEA (a): 4-pole machine, (b): 8-pole machine, (c):12-pole machine, (d): 24-pole machine.

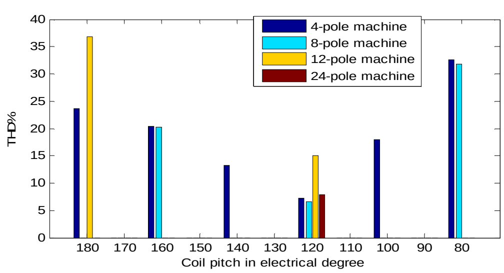


Fig. 8. THD% in phase EMFs with different coil pitches by FEA.

V. Comparisons

A. Different Coil Pitches

Fig. 8 shows the THD% comparisons of the machines and it can be seen when the coil pitches decrease from 180° to 120° electrical degree, the THD of all the machines also decreases, afterwards they will increase again. This is because the third harmonics in the phase EMFs are the mainly dominant ones after the fundamental harmonics. For symmetrical 3-phase machines, the triple harmonics can be cancelled out by star connection and hence will not present in the line EMFs. It is therefore usual to select the coil span to reduce as much as possible the 5th and 7th harmonics.

B. Distributed Windings and Concentrated Windings

Comparing Fig. 7 (a) with (c), which are respectively the harmonics of the 4- & 12-pole machines having integral-slot windings, the unwanted low-order harmonics of the distributed winding are suppressed much more than those of the concentrated winding when their coil pitches (in electrical degree) are same. Comparing the Fig. 7 (b) and (d), which is individually the harmonics of the 8- & 24-pole machines having fractional-slot windings, we can also find out that the distributed winding reduces more harmonics than the concentrated ones when the coil pitches are same. This is more clearly shown by the THD% comparisons in Fig. 8.

C. Integral-Slot Windings and Fractional-Slot Windings

The winding factors of the 4-pole machines with the coil pitches of 8/9, 6/9 and 4/9 are respectively the same as those of the 8-pole machine with the coil pitches of 4/4.5, 3/4.5 and 2/4.5 according to (4)-(7), So do the 12- and 24-pole machines when they have the coil pitch of 2/3. The difference is that the 4-pole and 12-pole machines have integral-slot windings whilst the 8-pole and 24-pole machines have fractional-slot ones. Theoretically, in fractional-slots windings, the ripples generated in one phase-group winding are out of phase with those in other groups and therefore the EMFs should have less harmonic content than in integral-slot windings. The possible induced slot harmonics in an electrical machine are mainly $2mq\pm 1$ orders [2]. Here there are the 17th & 19th for the 4-pole machine and 5th & 7th for the 12-pole machine.

Fig. 9 presents the comparison of the THDs in the EMFs of the 4- and 8-pole machines from both the FEA computations and experimental measurements. It shows that the harmonic content of the two machines are almost at same level. This is also verified by the experimental results presented in Fig. 15 and Fig. 16, where the measured EMF waveforms and their harmonics are provided. The reason is that the slot harmonics are very small in the 4-pole machines as shown in Fig. 7 (a), where the 17th and 19th harmonics are negligible compared to the low orders, so the comparison can not effectively shows the slot harmonic reduction by the fractional winding of the 8-pole machine.

Fig. 10 shows the comparison of the THDs of the 12- and 24-pole machines. The latter is almost the half of the former. The reasons of explaining this are follows:

1. The slot harmonics of the 12-pole machine are the 5th and 7th, which might be relatively large due to the concentrated winding construction.

2. The airgap flux density distribution of the 24-pole machine as shown in Fig. 11 is more sinusoidal than to be square due to the magnetic flux leakage between the magnets. And the slot effect in Fig. 11 can be effectively reduced by the fractional-slot winding.
3. The 12-pole machine has two magnets per pole, which introduces extra harmonic in the EMFs of the machine.

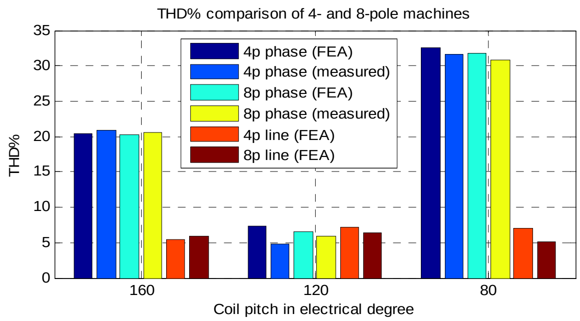


Fig. 9. THD% comparison of the 4- and 8-pole machines.

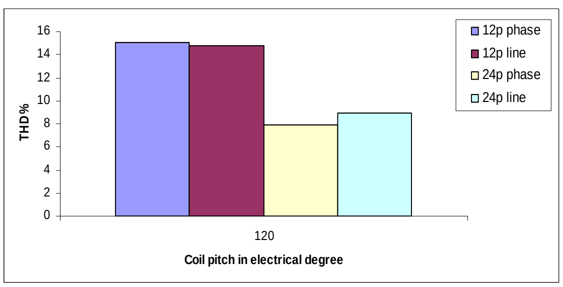


Fig. 10. THD% comparison of the 12- and 24-pole machines.

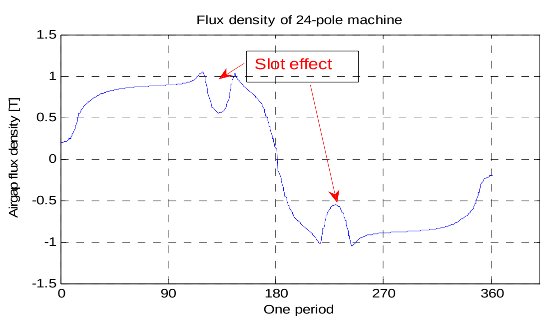


Fig. 11. The airgap flux density of 24-pole machine at certain instance.

VI. Laboratory Setup and Measurements

The 4-, 8- & 24-pole prototype machines are made in the laboratory. The 12-pole machine having integral-slot concentrated winding, which is not interesting for winding study, is not chosen.

The prototypes have open stators and two copper bars in each slot with connection terminals outside as shown in Fig. 1. The end winding connections are re-connectable. Just by varying the connections, different winding patterns can be obtained for experimental tests.

Fig. 12 shows one set of the test machines. The prototype machine is driven by an induction motor, which is controlled by a frequency controller. For the sake of safety, the maximum speed is limited to 1000 rpm.

Fig. 13 and Fig. 14 respectively show the comparisons of the measured EMFs and those from the FEA computations of the 4- & 8-pole machines with some coil pitches. The measured waveforms match the FEA waveforms quite well.

Fig. 15 presents the measured EMFs both of the 4-pole machine and of the 8-pole machine and Fig. 16 shows their harmonics. They are very similar as mentioned previously.

The induced phase EMF and line EMF of the 24-pole machine are also measured and they are satisfactory with the FEA results as illustrated in Fig. 17 (a) and (b).

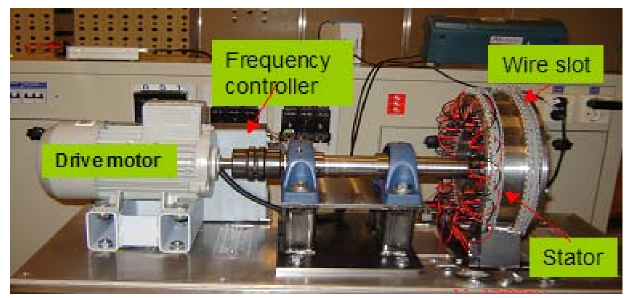


Fig. 12. Laboratory setup.

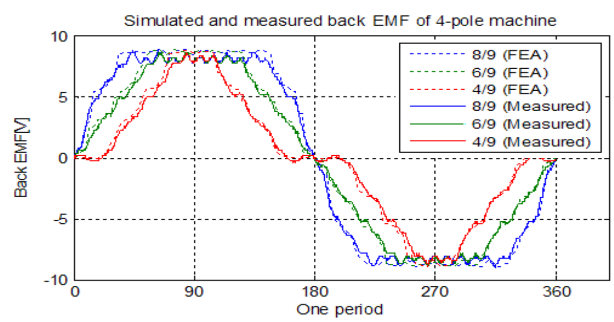


Fig. 13. Back EMFs from FEA and measurement of the 4-pole machine.

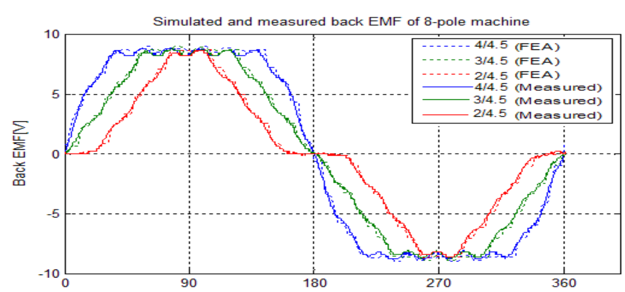


Fig. 14. Back EMFs from FEA and measurement of the 8-pole machine.

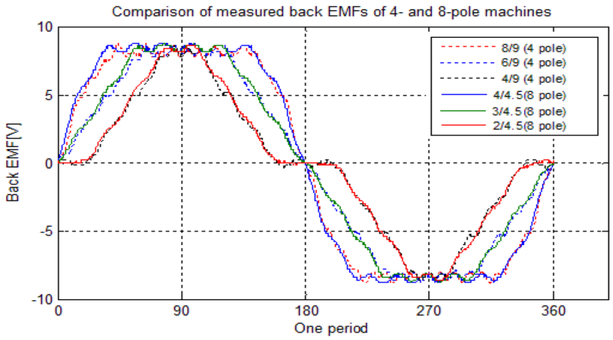


Fig. 15. Measured EMFs of the 4- and 8-pole machines.

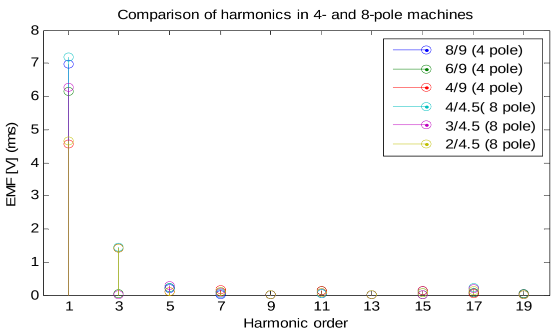
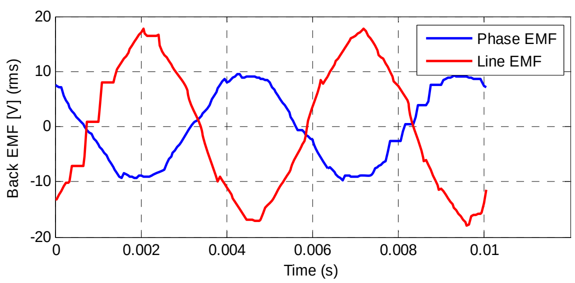
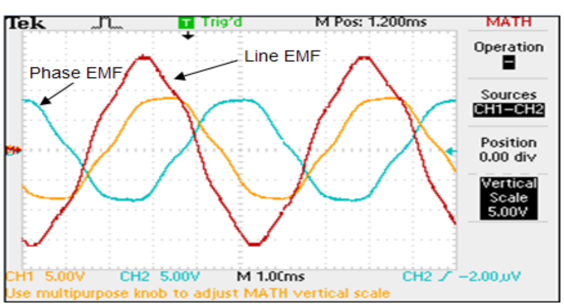


Fig. 16. Harmonics in the measured EMFs of the 4- and 8-pole machines



(a)



(b)

Fig. 17. Phase andlLine EMFs of the 24-pole machine
(a) by FEA, (b) by measurement.

VII. Conclusions

This paper has analyzed the back EMFs of four surface-mounted PM machines based on FEA computations. They are *a*): a 4-pole machine with integral-slot distributed winding. *b*): an 8-pole machine with fractional-slot distributed winding. *c*): a 12-pole machine with integral-slot concentrated winding and *d*): a 24-pole machine with fractional-slot concentrated winding. And three prototypes of *a, b* and *d* are made in the laboratory to verify the analysis results. Only considering the harmonic reduction, the conclusions are follows:

1. The integral-slot distributed winding is better than the integral-slot concentrated winding.
2. The fractional-slot distributed winding is better than the fractional-slot concentrated winding.
3. The fractional-slot distributed winding of the 8-pole machine does not effectively show the slot harmonic reduction compared with the integral-slot distributed winding of the 4-pole machine due to the relatively small slot effects in the machines.
4. The fractional-slot concentrated winding of the 24-pole machine is much better than the integral-slot concentrated winding of the 12-pole machine. For explaining this, here the flux leakage between the magnets in the 24-pole machine should be also taken into account.
5. When the coil pitches decrease from 180° to 120° electrical degrees, the harmonic contents in the phase EMFs of all the machines also decrease, afterwards they will increase again.
6. The prototype machines provide good opportunities for students to study different winding concepts and induced EMFs.

Acknowledgment

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